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CRUSHING OF THE TASKS IN MATHEMATICS EDUCATION AT VARIOUS EDUCATIONAL LEVELS

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ABSTRACT. *The subject of this paper are selected issues concerning some non-standard method of solving mathematical tasks. The method of crushing tasks (presented in the paper) is a way of working on the task which involves modifying the text of the task (providing more or less data, replacing the data, transforming the task, introducing new associations, etc.). The method can be used at every level of education.*

KEY WORDS: *mathematics education, solving mathematical tasks, crushing tasks*

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Introduction

Mathematical knowledge and skills have been playing a more and more important role in our daily lives. At the same time, solving tasks is the essence of mathematics understood as a field of human activity. One of the possible ways of looking at mathematics is that from the perspective of mathematical tasks. We can say that creating and solving problems is the major goal of mathematics at every stage of mathematics education. With the goal in mind to prepare students, in the course of the educational process, to living in the surrounding reality, one should emphasize the tasks which allow the pursuit of general objectives of mathematical instruction, i.e. those developing skills and attitudes necessary to a modern person, regardless of his or her field of activity. This goal may be reached by suggesting the *strategy of crushing* to the students.

The method of *crushing tasks* is a way of working on the task which involves modifying the text of the task (providing more or less data, replacing the data, transforming the task, introducing new associations, etc.). The name of the method is adopted from the French pedagogues A. Kaufmann, M. Fustier, A. Drevet. The researchers assume that deep immersion in the past and present inhibits the development of the future. And in fact, it is necessary to make room in the mind for new objects that do not exist yet but are about to come to being. We must, therefore, crush the existing objects to create new ones in their place (Hanisz, 1990).

The starting point in this method is the base task. "This is a task which is complex, open, and never contains questions" (Nowik, 2011). Themes underlying the task must be close to the student and somehow related to the interests of the person working on the task. The crushing method can be applied in different versions. J. Hanisz offers five versions of this method (1990).

Version I. Forming questions to the base task given by the teacher (What can be calculated?)
Step 1. Familiarizing students with the text of the base task;
Step 2. Forming questions to the base task (What can be calculated using the text and data of the base task?);
Step 3. Verifying the questions proposed by the students (If and how to calculate the unknown contained in the question?);

Step 4. Modifying the text of the base task (Select any question. Compose a word problem that fits it. Solve the problem).

Version II. Composing the mathematical formula fitting the base task given by the teacher (What will you calculate with this formula?)

Version III. Elaborating coded schemes for solving the base task.

Version IV. Modifying the base task given by the teacher according to the question: What would happen if…?

Version V. Posing questions and adding new data to the base task given by the teacher (see Jabłońska, 2011).

These versions of the crushing method do not exhaust all possibilities. Teachers and students can modify them or create new versions of the crushing method.

The first part of the paper presents a small piece of the research results. The second part presents a problem which can be considered at various levels of education.

Discussion of the research

Students were placed in a situation which was unusual for them. It was suggested that they would work on nonstandard tasks. The research methodology was based on the analysis of the solutions presented by the students. Additional research material included the students’ responses to the questions attached to the tasks. The research group consisted of 35 fourth-grade students of a Krakow primary school (eleven-year-olds) divided into five groups. All of them were students with average results in science.

The main aim of the study was to identify students' skills in solving and composing word problems. The research attempted to diagnose the fluency (i.e. Does the student stop at forming a single question, or is he or she capable of forming a series of questions?) and flexibility (i.e. Does the student change the direction of thinking, move from one thinking track to another as new relationships in the base task appear to him?) of the participants’ way of thinking. The study also aimed at answering the following questions. Are the tasks and questions created by the students original? Are the tasks and questions properly built in terms of language and mathematical construction?

Analyzing the results of the research, attention was paid to whether the students, answering the question of the given task, were using the tasks solved previously. The questionnaire contained a few math problems. In this paper, one selected problem is presented and discussed. It is the following task.

> Janek bought 7 pencils at PLN 2 in a shop, and Kasia bought 5 notebooks at PLN 4 each.
>
> Compose as many questions as you can to the base task.

The number of questions composed by each student ranged between two and fourteen. There were 241 questions composed by the students altogether (on average one student composed approximately seven questions). The students were very creative in the process of forming questions and justifying that a question could or could not be answered. Schemes of solving individual tasks were laid out (see step two of problem solving by Polya (1975)). When the students concluded that the task could not be solved, the appropriate data were provided. The participants of the study were very active and willing to participate in the discussion. They presented accurate arguments to convince the others of their reasons. The participants created various questions as well as formed questions which indicated deep analysis of the task's text. The questions proposed by the children were divided into groups. Below are selected groups with sample questions specific for each group in brackets:

1. Questions related to Janek (e.g. How much did Janek pay?)
2. Questions related to Kasia (e.g. How many items did Kasia buy?)
3. Questions related to Kasia and Janek (e.g. How much did they pay together? Who paid more?)
4. Questions related to other objects present in the task (How much was the notebook?)

Several questions formed by the students contained additional information, such as the amount of money the children (or one child) had (What would be the change of Janek if he had PLN 20?).

Analyzing the results, we can state that in almost all tested students (except two) fluent and flexible thinking was well developed. Students with average learning results do not stop at composing one or two questions, easily creating whole series of them. This may indicate that they notice more and more new relationships in the base task. Therefore, they move from one track of thinking to another. As for the linguistic and mathematical correctness of the tasks composed by the students, it can be concluded that, in spite of numerous language mistakes, mathematical errors occurred rarely.

Within the study, the students had to solve some of their own tasks. For most of them this did not pose major difficulties. The analysis shows that most students (60%) were not able to use their previous knowledge (use the solutions of the previous tasks). They completed all calculations each time from the beginning.

The sailor method

In the next part of the paper a (real) situation is presented, which can serve as a starting point to crush non-standard tasks at different levels of mathematical education.

> Let us consider a commonly used procedure to draw one person from a group of n people, called the sailor method. The participants stand in a row and simultaneously hold up at least one finger of their right hand. Then all the fingers shown are added up and the participants are counted up to this number, as in the counting-out game. The person on whom the sum has been reached is the selected one.

This is not a math task; it is rather a description of a certain real situation. Several questions arise here.

It is a fair selection? I.e. Are each person's odds of being selected equal?

Do the odds of selecting a person depend on the order in which participants have been lined up?

Will the odds of selecting a particular person change if we start the counting from someone else?

These are the first questions that can be formulated on the basis of the considered problem. Searching answers leads us to the way of finding the general solution by setting variables, solving the simpler cases first, and then generalizing the procedure in order to obtain general results.

Assume that the participants are arranged in a row and the person from whom the counting will begin has been determined. Thus, the set of people has been ordered. It can therefore be assumed that each person has an assigned number, signifying his or her place in the counting line. It seems natural to assume that the participants extend fingers randomly , independently from each other. These assumptions allow mathematization. Adopting them, we turn the situation into an experiment. The problem is general, so we can start with the simplest situation when two people are involved in the selection. Let us find the answer to the question: Is the selection of one out of two people using the sailor method fair?

All equally likely cases of holding up fingers by two people are presented in the table. The header row contains possible numbers of fingers extended by the first person, whereas the header column contains possible numbers of fingers extended by the second person. The fields on the crossing of line i and column j contain the sum $i + j$ of fingers held up by both people, if the first has extended i fingers and the second j fingers, ($i \in \{1,2,3,4,5\}$ and $j \in \{1,2,3,4,5\}$).

	1	**2**	**3**	**4**	**5**
1	2	3	4	5	6
2	3	4	5	6	7
3	4	5	6	7	8
4	5	6	7	8	9
5	6	7	8	9	10

Consider two events: A - the first person is selected, and B - the second person is selected. Out of the 25 equally probable results,12 correspond to event A, and 13 results to event B, so the second person is more likely to be selected. Hence, the selection is not fair.
Let us consider another question: Is the selection of one out three people by means of the sailor method fair?

We can proceed in the same manner as before. We present all equally likely cases of extending fingers by three people (125 cases) in a three-dimensional table. In this interpretation, a set of outcomes of the experiment consists of 125 smaller cubes forming together one big cube.

It is easy to conclude that the selection is not fair in this case either. The third person in the counting line is less likely than either of the remaining two (the difference of the odds is 1/125).

Let us now ask the question if a number n exists with which the selection by means of the sailor method is fair?

We can conduct the following reasoning.

When selecting one out of n people, the set of outcomes of extending fingers consists of all n-member sequences whose members belong to the set {1,2,3,4,5}. Each outcome is equally likely. There are 5^n outcomes altogether.
A necessary condition of a fair selection is that n is a natural power of 5. This implies that when n is not a power of 5, the selection with the sailor method is not fair.

The above reasoning leads to the following questions:

Is the selection out of five people fair?

Is the selection out of 25, 125, 625, ... people fair?

What can be said about the fairness of the selection when the participants can hold up from 0 (fist) to 5 fingers?

What can be said about the fairness of the selection when the participants can hold up from 1 to 2 fingers?

What can be said about the fairness of the selection when the participants can indicate a natural number from the set $\{1,2, ..., k\}$?

Answering each of the above questions requires formulating and solving the relevant probabilistic task. For this purpose, it is necessary to translate the non-mathematical problem into the language of mathematics, to construct the appropriate probabilistic model, to perform the relevant calculations and interpret the obtained numerical results. Finding answers to the questions and thus solving the subsequent tasks is a method of finding the problem's solution which leads from the consideration of specific cases to obtaining certain general results. Crushing of the tasks may take here several forms.

Conclusion

In conclusion it is worth noting that in mathematical education, the important students' activity of composing questions and problems should be considered. Such tasks allow students to learn more about the structure of the problem and allow them to develop mathematical activity, including creative activity.

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