

EXAMPLES OF GEOGEBRA COUPLED WITH AN INTERACTIVE WHITEBOARD TO ACHIEVE FRESH APPROACHES IN TEACHING MATHEMATICS

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ABSTRAKT. *V posledných rokoch sa spolu s technologickým pokrokom objavili nové možnosti a požiadavky, ktoré súvisia s vyučovacím procesom. V článku uvidíme príklady nových prístupov vo vyučovaní matematiky, pričom sa zameriame na použitie GeoGebry s kombináciou interaktívnej tabule vo vyučovacom procese.*

KEŤOVÉ SLOVÁ: *didaktika matematiky, matematika výukové, informačné a komunikačné technológie, nové technológie, GeoGebra, interaktívnu tabuľu.*

ABSTRACT. *With the great technological advances of recent years appeared in both new opportunities and new requirements related to the teaching-learning process. The article will shown examples of possible new approaches to teaching math offered combination use GeoGebra with interactive whiteboard in the classroom.*

KEY WORDS: *Didactics of mathematics, mathematics teaching, ICT, new technology, GeoGebra, interactive whiteboard.*

CLASSIFICATION: *D40, U50, Q64*

Introduction

With the great technological advances of recent years both new opportunities and new requirements related to the teaching-learning process have appeared. You can see that around the world, including Poland, teachers and other educational enthusiasts are enhancing the efficiency of the educational process, as well as increasing student involvement, and the use of activity aids which often introduce additional building on the achievements of ICT. Extending the teaching arsenal of modern technology, however, imposes new approaches to teaching and it is becoming necessary to update the process. The way this transformation occurs, however, will depend on the means of teaching.

Interactive Whiteboard - an intuitive way to control your computer

An Interactive Whiteboard is one of many educational media which is becoming increasingly popular in Polish schools. The directors of many institutions are attempting to have their classrooms fitted with such technology.

In practice, the interactive whiteboard is a device that (usually) resembles a large white board, and which allows interoperability through its connection to a computer and multimedia projector. You can compare it to a big screen that responds to touch. Thus, you can treat it as a device for input and output in an easy and intuitive way with the use of a computer. It gives you control of all the programs running on the computer using only a pen [1]. The user of an interactive whiteboard can, in addition to the computer screen, display any content (such as MS Word files, MS PowerPoint, web pages, photos, videos, etc.) and also use it to handle any running program. An Interactive Whiteboard also includes other functions including the possibility of taking notes, writing the curriculum, etc., but these aspects have been deliberately omitted in this article.

GeoGebra - a universal tool

Among a group of mathematics' teachers belonging to the enthusiasts TI, free software is gaining increasing popularity. One such project is the program GeoGebra [2], which can be applied to various areas of science such as mathematics, physics, geography or economics. GeoGebra is a mathematical tool for interactive presentation and examination of issues connected to geometry, algebra, analysis and statistics. It is designed for both teachers and students. It is therefore a kind of DGS combined with CAS. Characteristic for this project is that all objects identified in the program (such as curves, equations, functions, tables, sets of points, etc.) and their properties, are closely related. It gives a unique opportunity not only to look at an object from the perspective of the various branches of mathematics (eg, the circle as a geometric object on a plane set of points satisfying a certain property, the equation of a circle, etc.) but also the possibility of linking and the free passage from one property to other in terms of algebraic and geometric analysis. Such an approach allows the automatic adaptation to the needs of the problem or issue under consideration in every aspect, no matter what the nature of the object was imposed on it during its definition at the beginning. This is a very valuable opportunity that good teachers can use in their lessons.

New and interesting possibilities arise when the teacher, engaged in the teaching process, combines the comprehensive capabilities of a computer program such as GeoGebra with an easy system for use with an interactive whiteboard.

Interactive help

The teacher, in order to fully realize the objectives which are pursued in the teaching of mathematics, cannot skip any of its basic components, namely: the development of mathematical concepts, solving problems, conducting mathematical reasoning, creating mathematical language [3]. In each of them we can find a place for ICT. Below are examples of lessons aimed at developing a new mathematical concept, exploration properties and relationships of mathematical objects, analyzing problems in a given situation, solution and interpretation tasks (especially tasks requiring high precision design), implementation and interpretation of the claims and their leadership and development of mathematical language.

The experiment was based on a number of lessons conducted on the basis of interactive materials - work-sheets prepared in the form of GeoGebra files. Dynamic interactive teaching aids presented: definitions, theorems, proofs, tasks, applications of the theorems, the study of property and structures. Topics focused on the properties of plane figures. Students who participated in the experiment are from high school classes (16 years old). The main emphasis of this class is on mathematics, physical and information technology. During the operation, the daily use of information technology is in the broad sense of the word. The resource use - e-learning platform or Internet - as well as graphing calculators and various computer programs, which for these students is as natural as traditional classroom drawing on paper, supported the task.

Introduction of new concepts (definitions)

Students traditionally encounter definitions on paper, where they read verbal notation, formal language, and sometimes even analyze a drawing illustrating one of many situations. Materials created in the GeoGebra program depict an innovative look at existing solutions. During the experiment conducted with the students, a unique method for

introducing new concepts was met. This interpretation allowed the student to pay special attention to all the conditions required by a mathematical object. Simultaneous interactive geometric interpretation allows you to see many cases in a short time using any combination of the conditions of the object (Fig. 1). This ensures that the TI pupil will have many examples of objects etc. which will help them understand and learn more comprehensively than if they were to base all their knowledge on a couple of 2D examples from a book.

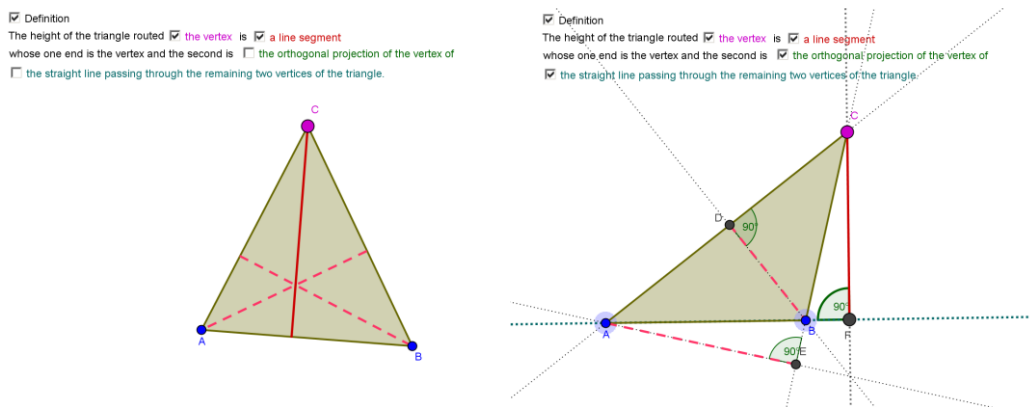


Figure 1 - Interactive definition of a triangle

That which is important in the case of interactive definition can also be a color, it is thanks to this that the student can relate the verbal condition included in the definition of a mathematical object or its property. Individual parts of this definition focus on showing the essential features of that object. The student can read the definition and actively participate in its creation.

In the opinion of students, a phrase which has been repeated many times is: this definition is "alive". There is only one opinion, but clearly you can see and extract the features necessary and sufficient for the designate as a given concept. Students emphasized that this approach affects the images which shape the concept and thus facilitates the search for counter-examples of the solution and its special cases.

Studying the concepts of ownership

Another aspect of the use of drawing as a dynamic interactive teaching aid is to study the properties of mathematical concepts. When students are faced with a problem they have to decide whether a given situation has met the definition, which often creates considerable dilemma when diagnosing the presented case. For example, there might be a problem in the construction of a particular type of triangle, e.g. an obtuse triangle (Fig. 2). Interactive help allows for easier imaging and makes suggestions for choosing arguments.

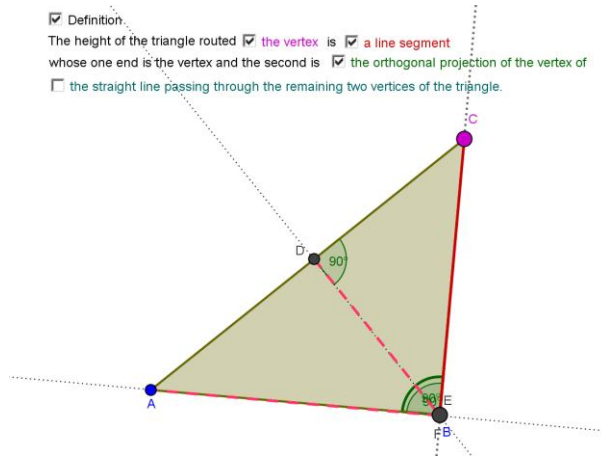


Figure 2 - Interactive definition of triangle - special case

In addition to resolving whether a situation is an example or perhaps a counter-example it is important to also study the properties of the observed objects and relationships in mathematics. One of the situations which were presented during the experiment was to investigate the position of the triangle, the orthocentre, depending on its type (Fig. 3).

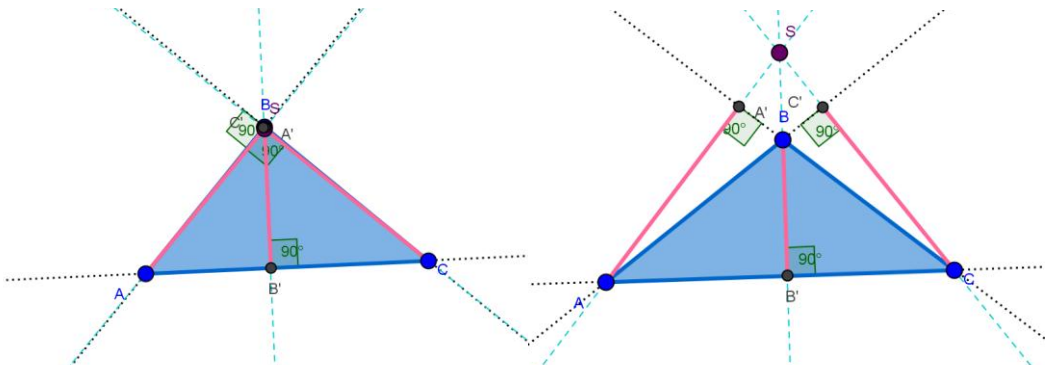


Figure 3 - Orthocenter of triangle - interactive simulation

Performed in the same way, lessons could be realized as an individual task, with discussion of the simulation, or otherwise. This stage of the definition introduced a combination, allowing the properties of newly-discovered objects to be found as well as preparing the groundwork for the formulation of new theorems resulting in a good field for problem-solving (this particular problem).

Students recognized this part of the experiment as most "active" and "creative". The ease with which they interacted with and manipulated the prepared materials meant that in a short time they were able to analyze a wide range of diverse examples, and to make classifications and formulate observations. A variety of examples facilitated students in gaining their arguments during the discussion, which was a great help.

Discovering and refining statements

This step may be preceded by a number of tasks (provoking) leading to the formulation of hypotheses. Formulating conclusions and making disciplines, however, can be problematic.

A new approach may help to refer matters when formulating proposals and claims. For example, the gradual introduction of further elements as assumptions and dependencies allow the observation of pertinent observations leading to the placing of hypothesis (Fig. 4).

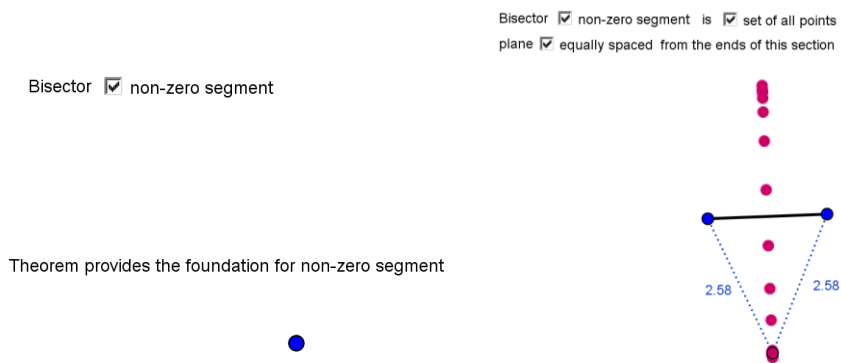


Figure 4 - Interaction and the discovery of theorems

Working in this way, the teacher takes care of the formal language in which observations are formulated as comprehensive and containing vocabulary of factual correctness. Furthermore, the teacher ensures that all necessary conditions have been included in the proposal, and draws attention to special cases. As with the definitions, the students were very enthusiastic about this new "theoretical" approach, which in most cases tied to the "rule with books." Here each of the formulated theorems allowed the observation of all the conditions and assumptions in specific instances. Many of the students emphasized that they preferred watching what happens to the object of mathematical analysis when such conditions are not met as assumptions. Didactic 'spoiling' can lead to better awareness of the need for some assumptions or the essence of certain dependencies.

Theorem

Do not forget the necessity of proving hypotheses. How can I make the proof easier to understand? Interactive teaching aids utilized in the GeoGebra program allow the student to be led step by step through a formalized proof and encourages the student, at the same time, to actively participate. The traditional paper-pupil method immediately reveals the course of the whole proof. The solution proposed by applications in the program encourages students to analyze each step while focusing his attention on the current situation and the relationships presented in a given time. Another attractive aspect is that the evidence of situation drawings is not bound. When finding proof using interactive teaching aids it is possible to check many different combinations of "set" objects without losing the evidential reasoning (Fig. 5).

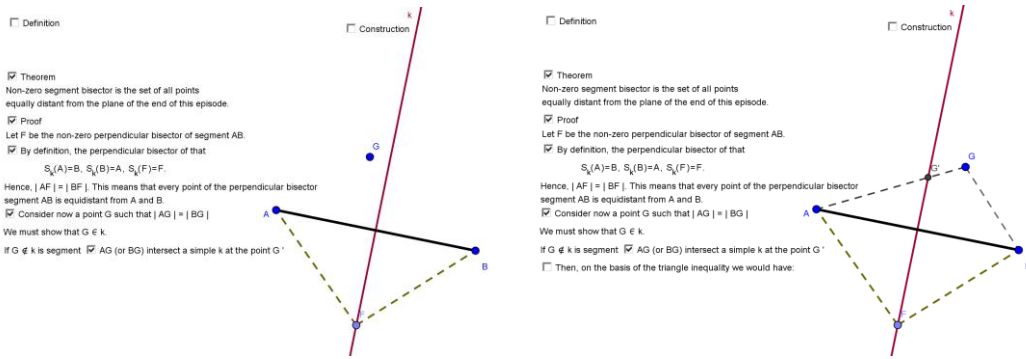


Figure 5 - Multi-stage interactive proof

Retaining evidence in such a way self-motivates students through the application of command executed in the program. An example of such student activity may be evidence that the length of the segment joining the sides of the harness means is equal to the arithmetic mean length of the base (Fig. 6).

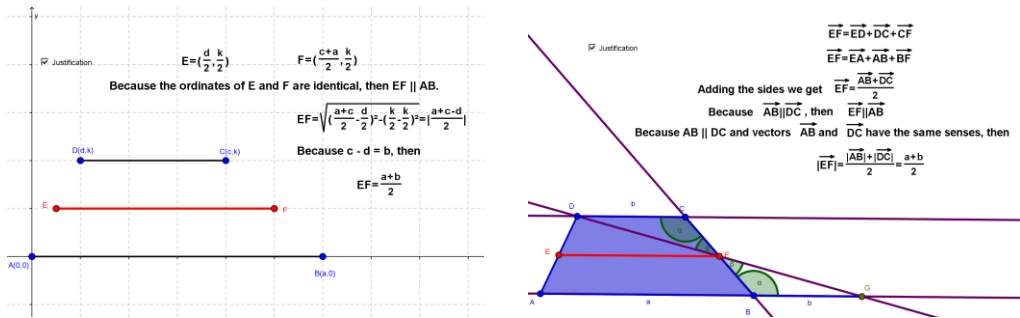


Figure 6 - Examples of evidence made spontaneously by students using GeoGebra

Students used the tools learned in the GeoGebra class. It replaced the tools with a piece of paper. Enrolled in the formal proof of the theorem, they made, on that occasion, an interactive drawing. In this way, an interactive document was created showing proof of the case, which an observer can change in any way. One student said, "Such a form I like, because the proof is formal, but made for a particular case. However, I can modify this case and see what will change. The proof, however, remains true in every case." This observation is particularly valuable, since many teachers have probably encountered situations where after the formal proof of a theorem in the general case the student was still not convinced of its truth for a particular case.

Conclusions and observations

Based on our experiment we can hypothesize that the transfer of knowledge using information technology in the broad sense of the term facilitates the assimilation of new knowledge, compared to traditional forms of communication. Consistent application of ICT as part of the learning process (in class) and learning/studying (eg at school, at home) meant that the students observed in class believe that a figure is not evidence, as well as simulation and testing of dozens of examples of the problem is not sufficient enough compared to recognition of the real observations. But it must be remembered that the group taking part in the experiment cannot be regarded as average. These are people who

definitely declare a positive attitude to ICT. The test performed indicated that these students would be happy to use the tool, not only for a deeper penetration of the substantive material but also as a tool for keeping notes, saving illustrations, discovering the logic, or observing his/her own conduct or progress. Constant use of skillfully-selected ICT changes the form of reasoning presented by the student or the solution. Students in this class reached for interactive visualization in the form of a file such as GeoGebra, as others have reached for drawings. However, it is a very valuable didactic instilling awareness that the ICT process of reasoning cannot be concluded that formal proof is required, whether it is the administration's argument or a counterexample. Students derive great help from the constant presence of modern technology in the classroom in the reinforcement of this approach.

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