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USING GRAPHIC DISPLAY CALCULATOR IN SOLVING SOME PROBLEMS CONCERNING CALCULUS

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ABSTRACT. *Calculus is considered as one of the most difficult topics in mathematics taught in high schools all over the world. A lot of students have problems with understanding the crucial objects like: limits of functions, geometric interpretation of derivative or even patterns of derivatives. It has occurred that ICT can help students muddle through this topic easily and helps them to understand it deeply. In this paper the role of graphic display calculator will be examined.*

KEYWORDS: *mathematics learning, limits, derivatives, graphic display calculator.*

CLASSIFICATION: *B10, C70, D40, Q60.*

Introduction

Teaching and learning mathematics in all levels is kind of a challenge for both teachers and students. As it appears some topics are more difficult than others. One of the most difficult topics taught in high schools is calculus, where majority of students have problems with understanding some crucial objects concerning this topic like: limits of functions, graphic interpretation of derivatives or even assimilation of patterns for derivatives. As a result, when students cannot understand such objects they do not want to learn further applications of differentiation. The consequence of such a situation may be an inability of studying some technical faculties at universities. Obviously not all syllabuses in high schools include this topic. However, one of such high school programs in which calculus is obligatory is International Baccalaureate Diploma Programme.

International Baccalaureate Diploma Programme (shortly IB) was established in 60s' of the last century in Geneva for gifted students aged 3-19. Nowadays International Baccalaureate Organization (shortly IBO) collaborates with about 3,500 schools in more than 100 countries all over the world. This program is divided into three groups: Primary Years Programme for children aged 3-12, Middle Year Programme for students aged 11-16 and Diploma Programme for students in age 16-19. As for this paper only Diploma Programme will undergo further consideration. (For further information you visit the Internet websites eg. ibo.org.pl).

Mathematics in Diploma Programme is divided into three levels:

1. mathematics higher level dedicated for students who want to continue their studies in technological universities or faculties closely deriving from mathematics,
2. mathematics standard level for students who are not to study mathematics directly but enroll in faculties using applications of mathematics (like economics, biology, chemistry, etc.)
3. mathematical studies standard level for other students.

Teaching and learning mathematics in all levels require their own syllabus but all of them are connected with using graphic display calculator which is a mandatory device used in all mathematical activities in this class and during final exams. Further information are to be found in [1] and [2].

Graphic display calculator (shortly GDC) was introduced to teaching mathematics in 90s' of the last century. At the beginning both teachers and students were skeptically as disposed to using this device. However, some researchers started to carry out their studies in which they showed that GDC could have positive influence on students while learning mathematics. Moreover, GDC could make this process more attractive for teachers and students. In the first decade of this century more and more researchers showed a significant impact of this tool on learning mathematics. For additional information concerning the usage of GDC during lessons, difficulties students had to face and problems while using it during public examinations (see [3], [4], [5], [6]). The researches were conducted in different fields of mathematics (not only for high school students) but we will focus only on some parts of calculus. Other papers concerning similar problems are to be found in [7], [8], [9]. Nevertheless, it seems that among these papers propositions of using GDC in finding limits and patterns for particular kinds of derivatives were not verified. Hence, my goal was to verify these propositions and find further applications of using GDC.

Prior to the research I asked the question

1. How can we use GDC in teaching students calculus?
2. How much teacher should engage himself/herself in teaching derivatives if he/she uses GDC?
3. Is it a good idea to use GDC in all activities concerning calculus?
4. What are the dangers of using GDC in these topics?

Although I was able to find a full answer for most questions asked, for some I have obtained only a partial answer.

Methodology and collecting data

The research was carried out among two groups of students attending IB class (higher level). The first group (named A) consisted of ten people (two girls and eight boys) whereas the second one (named B) consisted of 12 students (six girls and six boys). All students were taught by me. Students in both groups worked during normal 45-minute lessons. There were six lessons observed (four in the second group). Moreover, group B was in the first year whereas group A in the second year of IB class hence there were no similar tasks for both of them. In group A I carried out research tasks concerning limits of sequences and convergence of series. However, in group B I took into account the topics concerning limits of functions and patterns. In this paper I have limited my analysis to the second group only.

Each lesson had the same scheme. At the beginning of any given lesson I provided students with GDCs and tasks for solving. Additionally I introduced the topic but without any specific and particular propositions of solutions. During each lesson my help was restricted to solving problems with GDC itself (for example syntax errors). However, I did not propose or suggest any solution. Because the research was carried out during the initial lessons concerning the issue of differentiation and before the introduction of exponential and logarithmic functions and trigonometry I only considered polynomials and rational functions. However, the second type of functions I only considered in the first lesson about limits. During particular lessons students obtained such tasks to work with:

1. Finding limits of polynomials and rational functions.
2. Finding the patterns for derivatives of polynomials with different coefficients and degrees with at least two coefficients different than zero (the crucial was the pattern for $y = ax^n$ where a is a real number and n is a natural number which was the first part of this investigation).

3. Finding the patterns for derivatives of composite functions in the form:

$$y = (polynomial\ of\ degree\ k)^n$$

where k and n are natural numbers and examining roots of functions and their derivatives. Among others there were examples in which the inner function had real roots but there were also examples with complex roots only.

4. Examining monotonicity and concavity of polynomials by observing behavior of the first and the second derivatives.

Students during all lessons worked individually. Additionally, all lessons were recorded. During all lessons students used Casio fx – cg 20 with color and very precise screen.

Analysis

After the research I analyzed not only students' work papers but also recording. Since each lesson was concerned with another problem I took every lesson into account separately. It is worth to emphasize that in almost all tasks students used GDC mode Graph but some of them used Dyna Graph (during lesson 2) and Table (during lesson 1). However, it seemed that the mode Graph occurred the best one for this topic due to its visual aspects. Even though one can find Dyna Graph equivalent useful, in the aforementioned model of Casio GDC this mode works very slowly, hence it is rather not efficient enough. Below there is an analysis of each lesson respectively to the order presented in the previous paragraph.

1. Finding limits of polynomials and rational functions. During solving these tasks students who used only GDC mode Graph had no problems with finding proper limit which tended to the real number. However, students who used only Table mode were concentrated on the information of „no value" if the limit tended to a point in which the function was not defined. It is worth to note that almost half of students decided to use Table. Although none of them found answer no one checked the limits using mode Graph. Nevertheless, such problems did not occur when students considered limits of functions tending to infinity. In these kinds of examples students used only mode Graph because, as they emphasized in the interview, Table showed only finite number of values of the function. At this point we can sum up this lesson with an observation that students can understand sense of limits of functions but only when they use GDC mode Graph.

2. Finding the patterns for derivatives of polynomials. Students started their investigation with constant functions, next they proceeded to questions which were concentrated on linear, quadratic and cubic functions, but only in the form of monomials, i.e. $y = ax^n$ for $a \in R$ and $n \in N$. Students without any problems generalized the derivative of monomials, but more complicated examples of polynomials for which at least two coefficients were different than zero were rather difficult for almost half of students. Although students discovered the pattern for $y = ax^n$ they could not apply it for further polynomials, i.e. students could not discover patterns for sums and subtraction of monomials especially in the functions in which free term was equal to zero. We can sum up this lesson that without teacher's help the second part of the task was too difficult for students although they analyzed each pair of graphs simultaneously (the graph of function and its derivative on the same screen). Hence it is a good idea to use GDC to show students the relations between both graphs (of functions and corresponding derivatives). However, more complicated examples may confuse students.

3. Finding the patterns for derivatives of composite functions in the form

$$y = (polynomial\ of\ degree\ k)^n$$

where k and n are natural numbers. Students prior to this lesson were taught about composite functions especially where inner and outer functions are polynomials. In this part of the research their task was to recognize the patterns for derivatives of such a type of functions. During the whole work all student did not use any reduction formulas. They did not show for functions like polynomials but for composite functions only. Before this lesson they practiced calculating derivatives of polynomials. However during this lesson they were only concentrated on observing GDC screen. Additionally, students had different kinds of examples to solve. The first ones were when the inner function had real root(s). As a result students without any problems found roots which were common for the functions and their derivatives. Nevertheless, almost none of them found another zeros of derivatives which occurred in the derivative of composite function. The further examples in which the inner functions had no real roots did not help students. In this way students were not able to find general pattern for composite functions. To my surprise students had even problem with evaluating the degree of the derivatives of such functions. In this task GDC was not so helpful as before as it did not facilitate students to draw any generalizations.

4. Examining monotonicity and concavity of polynomials. The last lesson was the most difficult for students. Only four of twelve students tried to find similarities between the first derivative and the graph (examining monotonicity) and the second derivative and the function (examining concavity). (Some other students were concentrated on finding roots but they obtained wrong conclusions hence their work was no longer analyzed by me). Two of the four students finished their work successfully giving the proper theorems but without any assumptions. What is worth noticing is that students during the whole research knew only polynomials and rational functions and they did not know that there are functions which have no derivatives in particular points. In this research GDC was completely useless when students were unaided by the teacher as they had no prior knowledge what is the interpretation of the first and second derivatives. Without such information from the teacher students can find examining functions with GDC a waste of time

Further propositions

Further research was conducted in the same group of students. The research concerned integrations. This part of research will be presented very briefly due to the size of this paper.

Students' task was to find a general pattern for integrals while using GDC only. Because previous investigation focused on derivatives of polynomials, in this case students discovered general pattern for $\int ax^n dx,$ where a – real number and n –natural number. Students were unable to use mode Graph or any other mode because GDC can only calculate definite integrals. When I prompted my students, that they can consider $\int_0^x ax^n dx$ they quickly concluded the relation between differentiation and integration for other polynomials. They still did not understand the sense of the definite integrals and they completely confused when they started to examine $\int_0^x e^x\, dx$. They knew that $(e^x)' = e^x$ but the graph of $y = e^x$ and graph of its integral did not covered. Only they widened their investigations into the integral $\int_0^x e^x\, dx$ for $x \in \{-1, 0, 1\}$ they were able to understand sense of definite integrals and soon after they quickly created the pattern similar to Newton-Leibniz formula. However, it is crucial to emphasize at this point that students considered only polynomials and the functions $y = e^x$. Other examples have been not considered yet (especially trigonometric functions). In this topic GDC was very helpful for finding some basic patterns of integration but in a restricted sense due to the fact that students could only observe definite integrals and they did not recognize the sense of undefined ones.

Conclussions

Students used GDC for a few months prior to the research and they were familiarized with modes Graph and Table hence they had no problems with using those tools efficiently. However they solved some tasks focused on generalizations (especially using graphs) but only a few students were able to do all tasks properly. The only risk is that they could be familiar with these topics somewhere earlier.

Generally, the research showed that students unaided could generalize patterns only for simple tasks. However, in more complicated examples they could not apply obtained patterns for further examples. It means that the problem is not connected with the proper usage of GDC but with mathematical skills needed for a process of generalization. Even though GDC came in handy in this task, it could not substitute the logical thinking.

In the interview conducted after the research students admitted that they were so preoccupied with using GDC itself that they did not find any other solutions. However, they would probably be able to find general patterns otherwise. Additionally, they seemed to believe that they probably would not discover the main theorems of calculus because they worked only on polynomials (and rational functions in the case of limits).

Although students had some problems with differentiation they hardly had any problems with generalization of patterns for integrals. It means that students have to have more stimuli in solving tasks.

Answering the questions set in the introduction we can conclude that

1. It is a good idea to use GDC mode Graph to introduce sense of limits of functions rather than mode Table (especially in points where the function is undefined).
2. GDC can be used for finding general patterns for derivatives of monomials but for polynomials with at least two coefficient different than zero it seems to be too difficult (without any aid from the teacher).
3. Examining some other behavior of derivatives occurred too difficult for the majority of students (especially in the field of monotonicity and concavity but also for derivatives of composite functions).

To conclude finally, it seems profitable to use GDC in topic of limits and calculus. However, a teacher should do it in moderation. Moreover, a teacher should be engaged in teaching derivatives and integrals with GDC too as it appears that students familiar with modern calculators, when unaided, have problems with the generalization and confirmation of previously obtained patterns.

This research has showed that there are some dangers of investigation calculus with GDC. Hence in some examples it is the main role of the teacher to correct improper thinking of the students in order not to develop bad habits among them.

In the nearest future the research is planned to be repeated among randomly chosen students from universities (who are to learn calculus) in order to confirm or rejects my current observations.

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