

## SOLVING LINEAR DIFFERENTIAL EQUATIONS IN MATLAB

### RIEŠENIE LINEÁRNYCH DIFERENCIÁLNYCH ROVNÍC POMOCO MATLABU

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**ABSTRACT.** *The present paper describes the fact that revised and pedagogically justifiable implementation of CAS systems (e.g. Matlab system) is a great contribution to teaching higher mathematics. It brings practical applications to mathematics teaching and thus enhances motivation and students' interest in the discussed topic.*

**KEY WORDS:** *linear differential equation, Matlab*

**ABSTRAKT.** *V článku poukážeme na skutočnosť overenú dlhoročnou praxou, že premyslené a pedagogicky zdôvodnené implementovanie systémov CAS (v našom prípade systému Matlab) je veľkým prínosom pri vyučovaní vyššej matematiky. Posúva jej vyučovanie viac k aplikáciám z praxe, čím sa zvyšuje motivácia a záujem študentov o preberanú problematiku.*

**KEŤOVÉ SLOVÁ:** *lineárna diferenciálna rovnica, Matlab*

**CLASSIFICATION:** *C 70, D 40, B 40*

#### Introduction

In this paper we show the examples of Matlab application to the solution of linear  $n$ -th order constant coefficient differential equations. The assigned differential equation is solved with the help of simple commands of the system Matlab. The examples can serve as a simple manual for students – only the basic knowledge of Matlab and the knowledge of mathematics on the level of the first form of bachelor study is sufficient.

During seminars we teach our students how to understand differential equations and be able to solve them. From our experience we know that if the students are shown a concrete application of differential equations in the field they study their interest in the defined problem is higher. However, these application examples are sometimes computationally demanding and hence time consuming. But if these examples are solved with the help of the system Matlab they can be used in the teaching process to enhance the motivation of students.

#### Computer algebra systems (CAS) and their place in teaching mathematics

Computer algebra systems (including Matlab) are strong computational and visual tools with wide practical application. Using of CAS systems in mathematics teaching at the university has both supporters and opponents.

Bernhard Kutzler from ACDCA (Austrian Centre for Didactics of Computer Algebra) speaks about it in a very pertinent way: *“There exist thousands of ways – good and bad – how to use CAS in teaching. Inappropriate approaches usually come from technical enthusiasts – teachers who use CAS just because they exist. But CAS should never manage mathematics teaching – mathematics teaching (didactic intention) should manage their application.”* [4].

Helmut Heugl, the director of an Austrian project Derive and TI-89/92 adds: “*If CAS application is not pedagogically justified then it is pedagogically justifiable not to use it*”. [5].

Systems CAS were created to make routine, long-winded and complicated calculations instead of people, to make their work easier and provide desired results. But their usage only for this purpose is from a didactic viewpoint (when we emphasize the development of students’ mathematical education) insufficient and inappropriate.

We cannot be satisfied with the fact that our students are able to obtain the result by application of suitable mathematical software. Our ambition is to lead students to mathematical terms understanding and subsequent application of the received knowledge in practice.

Therefore we consider using CAS with comprehension to be the most appropriate one. In the first phase of the educational process we want our students to manage the discussed topic in a theoretical way and to learn the principle of the solution of specific exercises. This will help them to obtain a critical detached view necessary for the correct choice of a solution method and interpretation and evaluation of the obtained results. Only theoretically educated students are offered to use mathematical software suitable for the solution of more complicated application exercises.

We do not want to replace a traditional mathematical teaching by using a Matlab system. However we would like to motivate students to be able to solve also more complicated exercises from practice.

Matlab possesses a powerful library of functions serving for the solution of ordinary differential equations. What is more, Matlab has also extensive graphical possibilities which can be used in a teaching process. Ordinary differential equations, which often occur in engineering practice, are linear  $n$ th order constant coefficient differential equations.

### Basic terms

Linear  $n$ -th order constant coefficient differential equations have the form

$$y^{(n)} + a_1 y^{(n-1)} + a_2 y^{(n-2)} + \dots + a_{n-1} y' + a_n y = f(x),$$

where  $a_1, a_2, \dots, a_n$  are real numbers,  $f(x)$  is a continuous function on the interval  $\langle a, b \rangle$  and  $y$  is an unknown function. This equation can be put in the form

$$L(y) = f(x),$$

where  $L(y) = y^{(n)} + a_1 y^{(n-1)} + a_2 y^{(n-2)} + \dots + a_{n-1} y' + a_n y$ .

A general solution of this differential equation is searched with the help of the following theorem:

**Theorem.** Let  $Y(x)$  is a solution of a linear differential equation with the right side  $L(y) = f(x)$ . Then each solution of this differential equation is in the form

$$y = z + Y,$$

Where  $z = c_1 y_1 + c_2 y_2 + \dots + c_n y_n$  is a general solution of the corresponding linear differential equation without a right side  $L(y) = 0$  and  $c_1, c_2, \dots, c_n$  are arbitrary real numbers.

Generally, the solution  $Y$  can be found by the method of variation of constants.

**Theorem.** If  $y_1, y_2, \dots, y_n$  is a fundamental system of solutions of the differential equation  $L(y)=0$  then the solution of a linear differential equation with the right side  $L(y)=f(x)$  is in the form

$$Y = \sum_{i=1}^n y_i(x) \int \frac{W_i(x)}{W(x)} dx,$$

where  $W(x)$  is the Wronski determinant and  $W_i(x)$  is a determinant which results from Wronski determinant by replacing  $i$ -th column by the column of elements  $0, 0, \dots, f(x)$ . [3]

### Examples of solved exercises

With the help of Matlab we can solve a differential equation symbolically by using the command `dsolve`. Its application is presented in the following exercises:

**Exercise 1.** Let's find a general solution of the differential equation

$$y'' - 7y' + 10y = -(6x+7)e^{2x} \quad (1)$$

Let's find a solution of this differential equation that satisfies initial conditions

$$y(0) = 0, y'(0) = 1 \quad (2)$$

**Solution.** We solve a linear second-order constant coefficient and special right side differential equation. For its solution we use a command `dsolve` where we put the differential equation in apostrophes. A letter `D` indicates a derivative of the function. Generally notation `Dny` means  $n$ -th derivative of the function  $y = y(x)$  of the variable  $x$ . So `Dy` means the first derivative of the function and `D2y` means the second derivative of the function  $y = y(x)$  of the variable  $x$ .

```
>> y=dsolve('D2y-7*Dy+10*y=-(6*x+7)*exp(2*x) ','x')
y =
exp(2*x)*C2+exp(5*x)*C1+x*(3+x)*exp(2*x)
>>
```

The second way how to set a differential equation is its defining with the variable called `dr`. This way is useful if we solve more differential equations.

```
>>dr='D2y-7*Dy+10*y=-(6*x+7)*exp(2*x)';
>> y=dsolve(dr, 'x')
y =
exp(2*x)*C2+exp(5*x)*C1+x*(3+x)*exp(2*x)
>>
```

Thus a general solution of the differential equation (1) is

$$y = x(x+3)e^{2x} + c_1 e^{5x} + c_2 e^{2x},$$

where  $c_1, c_2$  are arbitrary real numbers.

Let's find the solution of the differential equation (1) which satisfies initial conditions (2). The differential equation (1) is defined by the variable called `dr`, initial conditions (2) will be defined by the variable `zp`. We also use a command `pretty` with the help of which a symbolic expression of a solution in the form similar to a mathematical notation is noted.

```
>>dr='D2y-7*Dy+10*y=- (6*x+7) *exp (2*x) ' ;
>>zp='y (0) =0, Dy (0) =1 ' ;
>> y=dsolve (dr, zp, 'x' )
y =
2/3*exp (2*x) -2/3*exp (5*x) +x* (3+x) *exp (2*x)
>>pretty (simple (y))
```

```
2/3 exp (2 x) - 2/3 exp (5 x) + x (3 + x) exp (2 x)
>>
```

Thus, the solution of the differential equation (1) satisfying initial conditions (2) is

$$y = x(x+3)e^{2x} - \frac{2}{3}e^{5x} + \frac{2}{3}e^{2x}.$$

With the help of the following commands we draw a graph of the solution on the interval  $\langle 0,1 \rangle$ .

```
>> x=linspace (0, 1, 20) ;
>> z=eval (vectorize (y)) ;
>> plot (x, z)
>>
```

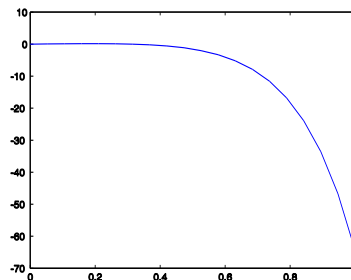


Figure 1.

**Exercise 2.** Let's find a general solution of the differential equation

$$y'' + y = \frac{1}{\cos^3 x} \quad (3)$$

**Solution.** We solve a linear second-order constant coefficient differential equation. Let's define it with the help of a variable called `dr`.

```
>> dr='D2y+y=1/ (cos (x) ) ^3 ' ;
>> y=dsolve (dr, 'x' )
y =
sin (x) *C2+cos (x) *C1+1/2* (1-2*cos (x) ^2) /cos (x)
>>
```

Thus a general solution of the differential equation (3) is

$$y = \frac{1}{2 \cos x} - \cos x + c_1 \cos x + c_2 \sin x,$$

where  $c_1, c_2$  are arbitrary real numbers.

**Exercise 3.** For an electric circuit in Figure 2 it is true that

$$RCL \frac{d^2 i_2}{dt^2} + L \frac{di_2}{dt} + Ri_2 = E(t). \quad (4)$$

Let's find a general solution of this differential equation if  $L=1 \text{ mH}$ ,  $R=10 \Omega$ ,  $C=1 \mu\text{F}$  and  $E(t) = 2 \sin(100 \pi t)$ . [1]

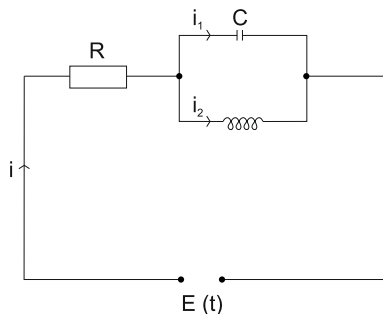


Figure 2

**Solution.** We solve a second-order linear constant coefficient differential equation. From the example it is obvious that solving this type of differential equation is time-consuming. Let's define a differential equation for a variable called **dr**.

```
>> dr='10*D2i2+Di2+10*i2=2*sin(100*pi*t)';
>> i2=dsolve(dr, 't')
i2 =
exp(-1/20*t)*sin(1/20*399^(1/2)*t)*C2
+exp(1/20*t)*cos(1/20*399^(1/2)*t)*C1
+(sin(100*pi*t)-10000*pi^2*sin(100*pi*t)
-10*cos(100*pi*t)*pi)/(5-99500*pi^2+500000000*pi^4)
>>
```

Thus a general solution of the differential equation (4) is

$$i_2 = \frac{(1 - 10000 \pi^2) \sin(100 \pi t) - 10 \pi \cos(100 \pi t)}{5 - 99500 \pi^2 + 500000000 \pi^4} + c_1 e^{-\frac{1}{20}t} \cos \frac{\sqrt{399}}{20} t + c_2 e^{-\frac{1}{20}t} \sin \frac{\sqrt{399}}{20} t,$$

where  $c_1, c_2$  are arbitrary real numbers.

## Conclusion

In several exercises of this paper the possibility of Matlab implementation in linear differential equations teaching was presented. Matlab system is a tool not only for elimination of routine and mechanical calculations but predominantly for enhancement of students' motivation during the process of this thematic unit learning.

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