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EXPRESSIONS WITH VARIABLES AND EQUATIONS IN ELEMENTARY SCHOOL THROUGH SOLVING OF REAL-LIFE PROBLEMS

VÝRAZY S PREMENNOU A ROVNICE NA ZÁKLADNEJ ŠKOLE PROSTREDNÍCTVOM RIEŠENIA PROBLÉMOV BEŽNÉHO ŽIVOTA

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ABSTRACT. *The article is focused on a research of appropriate procedures in introduction of concepts of variable, expression with variable and equation in elementary school. Problem situations with real-life context whose mathematical models are created with help of the concepts mentioned are formulated.*

KEY WORDS: *variable, expression with variable, equation, problem based task*

ABSTRAKT. *Článok je zameraný na skúmanie vhodných postupov zavedenia pojmov premenná, výraz s premennou, rovnica na základnej škole. Formulujú sa také problémové situácie bežného života, ktorých matematické modely sú vytvorené pomocou uvedených pojmov.*

KĽÚČOVÉ SLOVÁ: *premenná, výraz s premennou, rovnica, problémová úloha*

CLASSIFICATION: *C70, D40, D50, U40*

Introduction

Teaching of mathematics should lead to building the relationship between mathematics and reality, to gaining experience with mathematization of a real-life situation and with creating of mathematical models. Not very impressing results of Slovak pupils' assessment in international PISA measurements and the new state educational program encourage mathematical education from elementary school to be more oriented on development of knowledge and skills in solving of real-life situation problems, which need to be imposed and formulated. These requirements are quite well met by the latest mathematical textbooks for grades 5 – 8 by authors J. Žabka, P. Černek [5]. However, there is never enough of good and interesting practical problems. Therefore it is necessary to create collections of such problems, proven and taught in real elementary school conditions ([1],[2],[3]). Such problems are then widely applied in integration in the mathematical education process. This contribution is oriented on formulation of such real-life context problem situations whose mathematical models are created using concepts of variable, expression with variable, equation.

Expression with Variable and Equation with the Unknown in Elementary School

The concept of expression with variable is more-less used only in elementary school in propaedeutics of dependencies (functions) and equations with one unknown. As such, it is not featured in mathematical terminologies ([4]). But the truth is it is connected with functions. This concept is systematically prepared with use of the concept of expression with variable and dependency of two values. The concept of expression with variable is

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formed through problems (tasks) with comparing, containing phrases: “several more”, “several less”, “several times more”, “several times less”. Investigation in such problems leads to formal expressions with variable, e.g. $2 + d; 4 - T; 5x; 21Y; 3{,}7 + 1{,}5x$, where after substitution of a variable (d, T, x, Y) with a number we get a numerical expression, i.e. a problem to calculate. The result of this computation is called value of expression for a selected value, which we are looking for. Expressions with variable are then used to express the relation (dependency) between two variables (values). For example the notation

$$b = 2 + d, \text{ respectively } y = 3{,}7 + 1{,}5x$$

represents the dependency between values b and d, respectively y and x. The letter b respectively y may be seen as (mainly in initial simple tasks) as a name of expression $2 + d$, respectively $3{,}7 + 1{,}5x$. Often we want the value of expression to fulfill some requirement, e.g. to equal some previously given number. This is common when solving real-life situations, where we want the result to meet “our expectations”. This is how we come to the concept of equation, where the variable “becomes” the unknown. In general, we search for such number (numbers) that after substituting this number (numbers) in the equation two expressions with the same variable have the same value. Thus we search for an unknown number (numbers) with the said property. But now we already arrive to operations with expressions, equivalent modifications of equations.

Real-Life Context Problems with Models with Variable

Learning and teaching of mathematics can be made more attractive with the option of solving real problems with real-life context. This may result in positive attitude of pupils toward solving if such problem situations, as they may have already encountered or probably will encounter similar situations. This paragraph contains examples of such problem based tasks leading to creation of a simple mathematical model using expressions with variable.

Car Consumption

Cars have different average fuel consumption when driving in a city and when driving outside the city. For example Škoda Fabia has a quoted average extra urban consumption of 5,6 liters of fuel per 100 km and an urban consumption of 9,6 liters per 100 km.

Task 1. Fill in the following table of the average fuel consumption, if we are driving Fabia only in urban traffic.

km	50	100	150	215	x
consumption		9,6			

[$9{,}6 \cdot \frac{1}{2} = 9{,}6 \cdot 0{,}5 = 4{,}8$; $9{,}6 \cdot \frac{3}{2} = 9{,}6 \cdot 1{,}5 = 14{,}4$; $9{,}6 \cdot \frac{215}{100} = 9{,}6 \cdot 2{,}15 = 20{,}64$; $9{,}6 \cdot \frac{x}{100}$]

Task 2. Mr. Rosina drove 1 000 km per month, while x km in the extra urban traffic. Let S denote the overall fuel consumption in liters per month. Depending on x, how many liters of fuel were consumed? State S as an expression with a variable x.

[Extra urban consumption = $5{,}6 \cdot \frac{x}{100}$; urban consumption = $9{,}6 \cdot \frac{1000-x}{100}$; consumption per month = $S = 5{,}6 \cdot \frac{x}{100} + 9{,}6 \cdot \frac{1000-x}{100}$.]

On Sale

Shops often offer their stocks on so called *sales.*

The advertising leaflet of MODERNA shop included among other things the offer that if a total price of a purchase is at least 250 euro you will get a 40% discount. There was no discount on purchases under 250 euros.

Mrs. Mercantile originally intended to make a purchase worth 75 euro. However, the 40% discount attracted her. She agreed with her friend Mrs. Coin that they would merge their finances. Mrs. Mercantile put in their collective funds 90 euro, Mrs. Coin 60 euro and they made their purchase together.

Task 1. Did the combined 150 euro funds of Mrs. Mercantile and Mrs. Coin suffice the combined purchase worth 250 euro with the discount?

Task 2. If both friends bought merchandise costing 125 euro, what discount in % had in reality Mr. Coin with regard to her 60 euro contribution to the collective fund?

[Mrs. Coin saved 125 – 60 = 65 euro. Pupils can count % from 125:
1% of 125 equals the number $\frac{125}{100}$, so 1,25; 50% of 125 equals $\frac{125}{100} \cdot 50$, so 62,5;
52% of 125 equals $\frac{125}{100} \cdot 52$, so 65; x % of 125 equals $\frac{125}{100} \cdot x$. We may also think like this: 1% of 125 is 1,25; then 65 is $\frac{65}{1{,}25}$ percent of 125 and $\frac{65}{1{,}25} = 52$.]

Task 3. How should Mrs. Mercantile and Mrs. Coin fairly compensate, if both friends bought merchandise costing 125 euro and a criterion for a fair compensation is that the offered discounts (achieved benefits) of both friends will be at the same ratio as the sums of money added, namely 3:2.

[Both had a 125·0,4 = 50 euro discount. Mrs. Coin will give Mrs. Mercantile a sum so that their collectively saved money is in a ratio 3:2.

Mrs. Coin will give Mrs. Mercantile	5 euro	10 euro	x euro
Saved money of Mrs. Mercantile	55	60	50+x
Saved money of Mrs. Coin	45	40	50-x
Saved money ratio	$55:45 = 11:9$	$60:40 = 3:2$	(50+x):(50-x)

Mrs. Coin will give Mrs. Mercantile 10 euro as compensation, what could have been guessed.]

Didactic note. The way of compensation in the task 3 was proposed and enforced by Mrs. Coin. The easiest fair way of compensation considering the purchase would be to contribute to their collective funds with the same amount of money. This would be achieved if Mrs. Coin gave Mrs. Mercantile 15 euro. The truth was that Mrs. Coin did not want to join this action. Therefore Mrs. Mercantile persuaded her by offering her to share the money in a way Mrs. Coin would propose. Mrs. Coin was a walking calculator. The way of compensation may be left upon pupils' discussions as part of task 4.

Saving

Banks offer a variety of interest-bearing financial products with different penalties for early deposit withdrawals. If the financial market is stable, the interest rate of deposit with a longer notice period is usually higher. However, the disadvantage is the longer commitment period.

Bank A provides a 3% interest rate with annual term deposit. In case of early deposit withdrawal up to 80 days since the deposit opening no interest is given, and in case of 80 days earlier withdrawal interests from the withdrawn sum are reduced by 80 days. Bank B offers an interest rate of 4% for annual term deposit. In case of earlier withdrawal the interest rates of the withdrawn sum changes in the following way:

Period of deposit from- to (number of days) interest rate

0 - 29	0,00%
30 - 89	0,50%
90 - 179	1,00%
180 – 269	1,50%
270 – 364	2,00%
365	4 %.

Task 1. Calculate the interests in banks A, B after every 60 days and after a year from the deposited sum of 10 000 euro.

[In the bank A, the interest per day equals $\frac{300}{365} = 0{,}8219$ euro and per *x* days equals (*x*-80).0,8219 euro except the withdrawals after 60 days and 1 year . In the bank B the interest depends not only on the number of interest-bearing days, which is not decreasing, but also on the changing interest rate.]

x	60	120	180	240	300	360	365
A	0	32,88	82,19	131,50	180,82	230,13	300
B	8,22	32,88	73,97	98,63	164,38	197,26	400

Task 2. Express the difference between interests in the bank A and in the bank B depending on the number of days since the deposit of 10 000 euro from 90 days to 179 days. Decide about the profitability of making a deposit in both banks for 90 and 179 days.

[Interest in the bank A after x days equals $(x - 80) \cdot 0{,}8219$ euro, while we can substitute x with a random number from 90 to 179. Interest rate in the bank B for the deposit period 90 – 179 days is unchanged and equals 1%. Therefore interest rate in the bank B for every interest-bearing day equals $\frac{100}{365}$ eur $=$ 0,2739 eur.

After x days the interest in this bank equals $x \cdot 0{,}2739$ euro, while we can substitute x with a random number from 90 to 179. The difference between interests in the bank A and the bank B depending on the number of days of deposit input from 90 to 179 days is expressed by the expression $(x - 80) \cdot 0{,}8219 - x \cdot 0{,}2739$. If $x = 90$, the difference equals $8{,}22 - 24{,}65 = -16{,}43$. That means, in case of withdrawal of the deposit after 90 days, it is more beneficial to have the deposit in the bank B. If $x = 179$ then the difference equals

$$(179 - 80) \cdot 0{,}8219 - 179 \cdot 0{,}2739 = 32{,}34.$$

So in the case of withdrawal of the deposit after 179 days it is more beneficial to have the deposit in the bank A.]

The „Clever“ School

The primary School on the Clever street is famous for its talented pupils. This year there are also interesting numbers of pupils in classes at the lower grade. There are two classes at grade 1: IA and IB, and one class at each one of grades 2, 3 and 4. There are 16 pupils in the I.A class, 7 boys and 9 girls, and 24 pupils in the class at grade 2.

Task 1. We know that if four pupils from the I.A class would move to the I.B, it would be exactly two times fewer pupils in the I.A than in the I.B. What is the number of pupils in the I.B?

[There is intentionally given unnecessary information (7 boys and 9 girls in the I.A). If we denote by *x* the number of students in the I.B, the task leads to an equation $12.2 = x + 4,$ where we get the solution $x = 20$. However, beware of the false equations $\frac{12}{2} = x + 4,$ or $12 = 2.(x + 4)$, which often appear in pupils´ solutions. This task can be also easily solved by judgment.]

Task 2. The sum of pupils in the I.B and the I.A reduced by the proportion of pupils in I.A and the smallest even-integer indicates the number of pupils in the grade 3 class. What is the number of pupils in the third class?

[Since the smallest even integer is 2, the number of students in the grade 3 class is $(16 + 20) - (16/2) = 28$.]

Task 3. We know that the total number of grade 3 and grade 2 pupils is 16 more than the number of all first graders. Furthermore, if 5 boys and 3 girls from fourth grade would not come to school, there will be just as many fourth graders as pupils in the grade 2. What´s the number of pupils in the grade 2 a what´s the number of pupils in the grade 4?

[Let *x* denote the number of pupils in the second grade. We solve the equation $x + 28 = (16 + 20) + 16$, where *x* is 24. Let´s find out what is the number of fourth graders. The information "If 5 boys and 3 girls from fourth grade would not come to school ...“ is irrelevant, dividing students into boys and girls is not important. Let *x* denote the number of pupils in the fourth grade. Eight of them will not attend school, so we solve the equation $x - 8 = 24$, where *x* is 32, which is the number of fourth graders.

Task 4. Let *x* be the number of students in the class I.A. Express the number of all pupils in the grades (1-4) (and simplify it).

[Number of pupils for each class is: 16 (I.A), 20 (I.B), 24 (2.), 28 (3.), 32 (4.). Every "following" class has 4 more pupils than the "previous" class (each number of students represents one member of arithmetic progression with difference of 4), so the number of all pupils is $x + (x + 4) + (x + 8) + (x + 12) + (x + 16) = 5x + 40$.]

Task 5. Now let *x* be the number of students in the second grade. Express the number of all pupils in the grades (1-4).

Computing with Temperatures

Absolute zero is the theoretically lowest possible temperature, at which all motion ceases and at the same time it is the null point of the Kelvin scale. In Celsius scale, which is more commonly used in Slovakia, the absolute zero has the value approximately -273,16 °C. Another scale, mostly used in the USA, is the Fahrenheit scale. A German physicist Fahrenheit took as the null point of this scale, i.e. 0°F, the lowest temperature he was able to measure during his experiment in 1724.

When converting units of these scales we use the following equations, where K denotes temperature in Kelvins, °F in Fahrenheit and °C in Celsius:

$$K = 273 + {}^\circ C$$

$${}^\circ F = \frac{9}{5}{}^\circ C + 32$$

Task 1. Three friends – Joe, Erica and Matt were given a physics homework to measure the outside temperature ten-times per day and to compute the average temperature for that day according to the measured data. To make this task more interesting, each one of them measured the temperature using a different scale. As it was a cold day in January, Joe measured the average temperature -8°C, Erica got as a result 264 K and Matt, who measured the temperature in Fahrenheit, got the average temperature of 14°F. What average temperatures did each one of these three pupils measured on the Celsius scale?

[Joe measured -8°C, so this result doesn't need further editing. Erica got the value 264 K. If we denote the resulting number of Celsius degrees by x, from the given equations we get: $264 = 273 + x$, and from this equation it is easy to find that $-9 = x$. So Erica measured $-9°C$ as the average temperature. Matt's result also require to be converted into Celsius scale, we obtain the equation $14 = \frac{9}{5}x + 32$, after the modification we have $-18 = \frac{9}{5}x$, and from this we get $-10 = x$. Thus Matt's average measured temperature equals -10°C.]

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