



THE DEVELOPMENT OF COMBINATORIAL SKILLS OF THE LOWER PRIMARY SCHOOL PUPILS THROUGH ORGANIZING THE SETS OF ELEMENTS

RADEK KRPEC

ABSTRACT. *The article is aimed at studying the organizational principles through the use of the preschool pupil and the lower primary school pupils within the scope of one experiment, and also at the forthcoming experiment with the upper primary school pupils. The experiment with the lower primary school pupils deals with their ability to spread a set of cards in such a way that they are able to memorize the layout. Within the scope of the experiment we observed not only the effectiveness and principles of the organization of cards, but also the originality of the layout. The other part of the article deals with the forthcoming experiment with the upper primary school pupils regarding combinatorial tasks solving.*

KEY WORDS: *organizational principles, combinatorics, organization of elements.*

CLASSIFICATION: B10

Received 18 April 2014; received in revised form 4 May 2014; accepted 6 May 2014

1. Introduction

Since 2012, one of the areas of our research is finding strategies for solving tasks related to combinatorics in lower and upper primary school. “Combinatorial thinking is based on the ability to organize the elements of the set into transparent charts, graphs, diagrams, and lists” ([3]). In pupils of lower primary school the development of the ability to organize set of elements is a propaedeutic to combinatorics. Those abilities are necessary to be developed on the lower primary education level ([1], [5]). This propaedeutic is properly worked out in the textbooks of mathematics for lower primary school by professor Hejný [2]. As far as the lower primary school pupils are concerned, during the experiment we observe their ability to organize sets of elements or sets of phenomena. Despite the fact that the teachers try to explain combinatorial thinking to the upper primary school pupils and the secondary school pupils, the teaching process tends to follow the pattern of solving tasks according to a particular algorithm: pupils choose the right formula, solve the tasks and deliver the answer. Therefore it is important to develop abilities to solve combinatorial tasks in the primary school [4]. The aim of our ongoing experiment with the upper primary school pupils is to observe solving strategies and the changes of the strategies for solving combinatorial tasks.

2. The Experiment with Lower Primary School Pupils

The experiment, which included one pupil in preschool age named Adélka, and two pupils of first year named Markéta and Marek, two pupils of second year named Klára and David, and one pupil from third year named Adam of the lower primary school, took place in March 2013. The pupils took part in the experiment individually. The goal of this experiment was to find out in which way pupils organize objects. During the experiment we observed some of the didactic parameters such as environment, used language, and mainly the organizational principle.

2.1 The Description of the Experiment

The experimentalist and one pupil take part in the experiment. The experimentalist has four sets of cards, each of them in two identical versions.

Set A. Six different animals: rooster, hen with chickens, monkey, bear, dog, hare.

Set B. Yellow cards with numbers: 1, 2, ... 11, 12 (See Figure 1).

Set C. {blue, yellow} $\times$ {all six sides of a die} (See Figure 2).

Set D. {blue, yellow, green} $\times$ {bear, monkey, rooster, dog}.



Figure 1: Set B



Figure 2: Set C

The game is optional, pupils can end it anytime and go to play.

2.2 The Schedule of the Experiment

Action 01. Introductory interview. The experimentalist gives a pupil the set A and says: “Here you have a set of six cards. Spread the cards on the table. Then, we will turn the cards upside down and you get a second set of cards, which you will be supposed to order in the same way”. The experimentalist gives the pupil the set A. In case the pupil says or asks anything, the experimentalist reacts accordingly. The pupil spreads the cards. A few seconds after the last card is laid on the table, the experimentalist asks: “Do you remember the sequence of the cards? Can we turn them upside down?” The experimentalist waits for pupil’s approval and turns the cards. If the pupil joins him/her in the process, the experimentalist thanks them.

Ex: “Here you have the second set (gives the pupil the second set). Spread it in the same way as the first one”.

The pupil spreads the second set of cards, the experimentalist remains silent. After the pupil finishes spreading the cards, the experimentalist asks him/her: “Do you think this is correct? Shall we start turning the cards, or do you want to change anything?” Ex: “Let’s turn the cards upside down.” The experimentalist turns the second set of cards upside down. The experimentalist evaluates the pupil’s result. If the result is successful (which is expected), the emotional part of the evaluation is intensified.

Action 02. The experimentalist repeats the Action 01 experiment, but turns the cards upside down immediately after pupil finishes spreading the first set (about which he/she notifies the pupil beforehand).

Action 03. The experimentalist pulls out two sets B and explains to the pupil that they will play the same game as with the set A. Moreover, if the pupil succeeds in spreading the two sets in the same way, he/she is given a point. If not, the experimentalist is given a point. The game and evaluation will take place. Other actions with the same set follow; now the pupil is expected to spread the cards in a different way. These actions are then repeated once or twice according to pupil’s needs and interest.

2.3 Didactic Parameters

During the experiment we observed several didactic parameters such as environment, used language, and mainly the organizational principle.

As far as an environment is concerned, we can choose from three options: a) semantic, b) structural, c) hybrid. As for the language, mainly the manipulative language was used (combined with the graphic language). Our main interest was the organizational principle. We looked for the organizational principle in all tasks, in fact for organizational principles, since several organizational principles might have occurred during a single action.

2.4 The Results of the Experiment

First we analyzed the set B experiment. We left out the training actions. Now we will gradually focus on experiments with pupils from the preschool age to the third year.

The preschool age pupil's name is Adélka. She spread the set in a creative way (she followed a pattern that enabled her to memorize the sequence). We can conclude that her layout is not influenced by working with numbers in lower primary school. She spread the set B as follows:

1	2	3	4	5
6	7	8	9	10
11	12			

The layout in the next action was again from 1 to 12, but in this case it was L-shaped.

The set C layout was no longer creative. The cards were spread in a way for her to remember the sequence. First the numbers from 1 to 6 in two lines, the first line being blue, the second line yellow. In the next game, she spread the blue cards into two lines, 1 3 5 in the first line, 2 4 6 in the second, and then she spread the yellow cards in the same way underneath the blue cards.

She spread the set D into the 4 x 3 pattern. She divided animals into lines and colours into columns.

The lower primary school pupils, Marek and Markéta from the first year, had different card spreading strategies. Marek chose the most space-saving layout. He spread the set B cards in ascending order or in descending order. Markéta, on the other hand, was more creative. She tried to spread the cards in a way to remember them correctly, but she also tried to add something that would make the layout more interesting or more demanding. As for the first try, she spread the cards as follows: 1 3 2 4 5 7 6 8 9 11 10 12. In this case, however, Markéta was not able to spread the second line in the same manner, as the end of it may become disrupted by double figure numbers, and she spread the line as follows: 1 3 2 4 5 7 6 8 9 11 12 10. In the following game she chose a less demanding strategy (even if it is still not completely easy) and spread the cards as follows: 1 2 3 4 5 7 6 8 9 10 11 12. We do not know if the switch of numbers in the middle of the line was intentional, or if it was accidental and she only remembered which two numbers she switched. She spreads the second set in the same way. As far as the set C is concerned, Marek's economy of the card spreading and Markéta's creativity once again become apparent. Marek always puts cards of the same colour together and orders them either in an ascending or in descending order within the scope of one colour (e.g. 6 5 4 3 2 1 1 2 3 4 5 6). Markéta spreads the cards in an ascending order from the edge to the center; odd numbers from the left, even numbers from

the right; yellow cards first then the blue ones: 1 3 5 1 3 5 6 4 2 6 4 2. She remembers this layout very well. Therefore, in the next game she chooses a more demanding strategy: 2 2 3 3 4 4 5 5 6 6 1 1. Even though the colours do not alternate regularly, Markéta is able to remember the sequence. In the first set of D game, Markéta once again aims for creativity, but is unable to remember the layout. Therefore, in the next game, she chooses an entirely economic strategy, similar to the one Marek chose in all set D games. The example of an economic layout:



Figure 3: Example of the economic strategy

The next ones who took part in the experiment were Klára and David – second year pupils. Once again, differences in the spreading strategies (organizational principles) are apparent. Klára chooses the most economic way of card spreading. As far as the set B is concerned, she chooses the sequence of 1 to 12 either in ascending or in descending order. David chooses much more demanding layout. In individual actions, he spreads the cards gradually and in pairs: e.g. 1 9 11 2 8 4 7 5 10 12 6 3. When the second spreading comes, the problem occurs with remembering the sequence of the pairs (e.g. 4 8) or with confusing the entire pairs. In the third game with this set, he chooses the layout that is easier to remember: 12 11 ... 2 1. As for the set C, Klára once again chooses the economic layout: the cards of the same colour are put together; there is regular alternation of colours in a line with numbers being lined up either in ascending or in descending order. David once again chooses not so much economic layout, the result of which is that he is not able to remember it. The example of his layout is as follows: 1 1 6 6 5 5 4 4 2 2 3 3. A similar situation occurs in other set C games. It is not before the final game that David chooses the ascending sequence of cards with regular alternation of colours. As far as the set D is concerned, Klára once again chooses the easy organizational principle: she puts the same animals together; and chooses either alternations of colours in lines or the Cartesian layout 3×4 . David too chooses the easy layout for this set.

The last one to take part in the experiment was Adam, a third year pupil. In the first set of B game, Adam uses the effective layout of cards. He spreads the cards into two lines: the first line having odd numbers in it, the second line having even numbers in it; numbers in both lines are lined up in ascending order. In the second game, however, Adam violates his organizational principles, the result of which is that he does not remember the first layout when doing the second one. Therefore, in the third game he chooses an entirely effective layout: puts all the numbers to one line and lines them up in ascending order from 1 to 12. In the first set of C game he once again uses a certain organizational principle for numbers, but uses no such principle for colours. This results in his not remembering the layout. The layout is as follows: 2 2 4 4 6 6 in the first line, and 1 1 3 3 5 5 in the second line with colours not repeating regularly. In the second game with this set Adam uses regular alternation of colours. The third set of C game is interesting. In this game Adam tries to use suitable organizational principles but at the same time tries to add creativity to the game. The layout is as follows: there is a line of in ascending order lined up yellow cards, which is interrupted by a column of blue cards lined up in descending order:



Figure 4: A line of in ascending order lined up yellow cards, which is interrupted by a column of blue cards lined up in descending order

In the set of D games Adam follows the organizational principles: the same animals in lines or columns, and the cards of the same colour in lines or columns. He uses either the 3×4 or 4×3 layouts.

2.5 The Summary of the Experiment with the Preschool Pupil and the Lower Primary Pupils

From the comparison of the experiments it is evident that the organizational skills used by pupils differ considerably in terms of structure, effectiveness, and difficulty. Some pupils (Marek, Klára) aim for the easiest (most effective) principle of card spreading; the one they can remember well. Other pupils (Markéta, David, Adélka, Adam) use the effective layout only after failures in the early games. Markéta chooses effective organizational principles into which she tries to add a creative aspect. This results in the violation of simple organizational principles. As far as David is concerned, we think that his failures are caused by his searching for suitable organizational principles, not by trying to make the game more interesting. Further we can notice, that while Markéta prefers to choose linear strategy in spreading the cards, Adélka and Marek with the set of C and D game choose mainly the strategy of Cartesian layout. At the beginning David keeps the linear layout but with the last set he chooses the Cartesian layout of cards, the same as Klára. The experiments prove that the pupils were able to find the organizational principles that helped them win the game every time. As the didactic parameters are concerned, from the used language mainly the manipulative in combination with graphic language occurred. The differences were mainly in the fact that Adélka, Markéta, Klára and David looked at the cards first and then they started to spread them on the table. On contrary Marek and Adam started to spread the cards immediately from hands, without looking at them first. That was one of the reasons why David had problems with finding a suitable organizational strategy. The environment was in this case hybrid, we worked on experience with the games of pairs type (semantic environment), the cards were adjusted for our experiment (structural environment).

3. The Experiment with Upper Primary School Pupils

Following the previous research, we will continue with upper primary school pupils. In this experiment we will deal with solving of combinatorial tasks. The research will have three phases. In the first phase, an experiment with individuals will be conducted. We will observe how their strategies change during solving of a combinatorial task, and how their solving strategies change with regard to solving the previous tasks. We will also be interested in which strategy of organizational principle they use to solve a combinatorial task. Besides, we will observe whether pupils are able to discover isomorphisms in solving of the individual tasks. In the next experiment, we will work with a group of pupils in a mathematical club. Apart from the aforementioned goals, we will observe the changes of

strategies of the pupils who watch another pupil solving the task (e.g. at the blackboard); and the change of strategy for solving a combinatorial task in a group. We will want to apply some of the experiments conducted in a group of pupils to the entire class.

4. Conclusion

The aim of these experiments is to find out how to prepare the lower primary school pupils so as they are ready to solve tasks of combinatorial character on the upper primary school level. As far as the upper primary school pupils are concerned, we intend to focus on the best way of preparing them for combinatorial tasks. The basis of the education process is a teaching method aimed at scheme building, which has been our main research area for the past several years. The research has been conducted in cooperation with the Department of Mathematics and Mathematical Education of the Faculty of Education of Charles University.

References

- [1] Haylock, D. H., Cockburn, A. D. 2003. *Understanding Mathematics in the Lower Primary Years*. London: Sage, 2003. ISBN 0-7619-4103-7.
- [2] Hejný a kol. 2009. *Matematika pro 1. až 5. třídu ZŠ*. Sada učebnic a pracovních sešitů. Plzeň: FRAUS, 2009.
- [3] Hejný, M. 1989. *Teória vyučovania matematiky 2*. Bratislava: SPN, 1989. ISBN 80-08-00014-7.
- [4] Scholtzová, I. 2006. Kombinatorika a budúci učiteľ matematiky. In: *Acta Mathematica 9*. Nitra: Univerzita Konštantína Filizofa, Fakulta prírodných vied, 2006. ISBN 80-8094-036-3.
- [5] Turner, S., McCulloch, J. 2004. *Making Connections in Primary Mathematics*. London: David Fulton Publishers, 2004. ISBN 1-84312-088-7.

Author's Address

RNDr. Radek Krpec, Ph.D.

Department of Mathematics with Didactics, Pedagogical Faculty, University of Ostrava, Mlýnská 5, CZ - 70103 Ostrava

e-mail: radek.krpec@osu.cz