



TESTING OF IRRATIONAL NUMBERS AT THE HIGH SCHOOL

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ABSTRACT. *This report is research on understanding of irrational numbers. Specifically, I focus here on how irrational numbers can be represented and how different representations influence participants' responses with respect to irrationality.*

KEY WORDS: *rational and irrational number, pedagogical experiment, research, understanding.*

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Introduction

Real numbers express quantitative properties of the real world. Therefore, the subject matter of real numbers included in mathematics textbooks for all types of schools. Studying at high school assumes mastering learning content in mathematics at a level that students could attend college. He should prove effective to acquire and process the information and mathematical form to real situations. He should understand the meaning and value of mathematics in the past and at present, get to know possibilities of application Mathematics in the Sciences and Humanities. Pupil relationship in high school being built through examples tackled.

The role of the teacher is to explain the given problem by a particular form to choose such tasks that attracted interest among students. I teach mathematics at university for 17 years. During my practice I found out that in explaining the subject matter is usually pupils meet with several sample solutions jobs. They then by copying these procedures solve tasks. Based on these observations I made the hypothesis, which I verified using the test.

Research setting

The hypothesis, I tried to verify by using the test. The main objective of these studies was to examine the knowledge of irrational numbers of high school students. I wanted to find out by testing as students distinguish rational from irrational numbers, operations with rational and irrational numbers, as well as understanding of the notion irrational number. For the purpose of experiment I randomly chose the group of 59 students from the 4-th grade of gymnasium from the Zilina region. The group was further divided into three groups. All the students were taught the same thematic unit.

Test

1. List the prime numbers from 1 to 30. Make of them irrational numbers.
2. Decide which of the following numbers are rational and the irrational:
 $\sqrt{0,25}$; $1,213\overline{213}$; π ; $3,21875143 \dots$; $\sqrt{150}$; $\sqrt{900}$; $\sqrt{0.28}$; $\sqrt{25}$; $7,3284317$;
 $2,\overline{11}$; $\sqrt{0,081}$; $\frac{\sqrt{20}}{\sqrt{5}}$.
3. Prove that the number $\sqrt{2}$ is irrational.
4. Which of the numbers $\sqrt[5]{5}$, $\sqrt{2}$ is greater?

5. Remove the square root of the denominator of the fraction $\frac{1}{\sqrt{3}-\sqrt{2}}$.

- The result is a) integer
 b) rational number
 c) an irrational number

6. Edit the expression $\sqrt{a^6b^4 - a^4b^6}$, at which $a, b \geq 0, a > b$.

Results and analysis of test

The test results are processed from the responses of students.

Test

1. List the prime numbers from 1 to 30. Make of them irrational numbers.

K – the set of prime numbers

Answer	Number of responses
1.1 : $K = \{1,2,3,5,7,11,13,17,19,23,29\}$ Irrational numbers: $\sqrt{2}, \sqrt{3}, \dots, \sqrt{29}$	7
1.2 : $K = \{1,3,5,7,11,13,17,19,23,29\}$ Irrational numbers: $\sqrt{1}, \sqrt{3}, \dots, \sqrt{29}$	6
1.3 : $K = \{2,3,5,7,11,13,17,19,23,29\}$ Irrational numbers: $\sqrt{2}, \sqrt{3}, \dots, \sqrt{29}$	17
1.4 : $K = \{1,2,3,5,7,11,13,17,19,23,29\}$ Irrational numbers: $\sqrt{1}, \sqrt{2}, \sqrt{3}, \dots, \sqrt{29}$	25
1.5 : $K = \{1,2,3,5,7,11,13,17,19,23,29\}$ Irrational numbers: can not be made	4

2. Decide which of the following numbers are rational and the irrational: $\sqrt{0,25}$; $1,213\overline{213}$; π ; $3,21875143 \dots$; $\sqrt{150}$; $\sqrt{900}$; $\sqrt{0.28}$; $\sqrt{25}$; $7,3284317$; $2, \overline{11}$; $\sqrt{0,081}$; $\frac{\sqrt{20}}{\sqrt{5}}$.

Answer	Number of responses
2.1 : correctly, reasoned e.g. π - has not completed decimal developing – the irrational number $\sqrt{900} = 30$ it is the rational number $2, \overline{11}$ - it is periodic - the rational number	19
2.2 : they did not know edit the number under the square root	13
2.3 : estimated, without justification	17

2.4 : they did not know to divide the numbers 10

3. Prove that the number $\sqrt{2}$ is irrational.

Answer Number of responses

3.1 : by using the Pythagorean Theorem 11

$$\begin{aligned} \sqrt{2} \in I \quad c^2 &= a^2 + b^2 \\ c^2 &= 1^2 + 1^2 \\ c^2 &= 2 \\ c &= \sqrt{2} \end{aligned}$$

3.2 : $\sqrt{2} = \frac{p}{q} \quad / ()^2$

$$2 = \frac{p^2}{q^2} \quad p, q - \text{relatively primes}$$

$$2q^2 = p^2 \quad 2/p \quad \wedge \quad p = 2a$$

$$2q^2 = 4a^2q^2$$

$$q^2 = 2a^2 \quad 2/q \cdot$$

3.3 : by using calculator 8

$$\sqrt{2} = 1,41 \dots$$

3.4 : they did not know 25

4. Which of the numbers $\sqrt[5]{5}$, $\sqrt{2}$ is greater?

Answer Number of responses

$$4.1 : \sqrt[5]{5} = 5^{\frac{1}{5}} = 5^{\frac{2}{10}} \quad \sqrt{2} = 2^{\frac{1}{2}} = 2^{\frac{5}{10}}$$

$$\sqrt[10]{5^2} \quad ? \quad \sqrt[10]{2^5}$$

$$25 < 32 \quad \text{then} \quad \sqrt[5]{5} < \sqrt{2}$$

4.2 : estimated, without a solution 24

4.3 : by using calculator 9

$$\sqrt[5]{5} \quad ? \quad \sqrt{2}$$

$$1,379 < 1,4142$$

$$\sqrt[5]{5} < \sqrt{2}$$

4.4 : they did not know 11

5. Remove the square root of the denominator of the fraction $\frac{1}{\sqrt{3}-\sqrt{2}}$.

The result is a) integer

b) rational number

c) an irrational number

Answer

Number of responses

$$5.1 : \frac{1}{(\sqrt{3}-\sqrt{2})} \cdot \frac{(\sqrt{3}+\sqrt{2})}{(\sqrt{3}+\sqrt{2})} = \frac{\sqrt{3}+\sqrt{2}}{3-2} = \sqrt{3} + \sqrt{2} \in I$$

23

5.2 :	$\frac{1}{(\sqrt{3}-\sqrt{2})} \cdot \frac{(\sqrt{3}-\sqrt{2})}{(\sqrt{3}-\sqrt{2})} = \frac{\sqrt{3}-\sqrt{2}}{3-2} = \sqrt{3} - \sqrt{2} \in I$	13
5.3 :	$\frac{1}{(\sqrt{3}-\sqrt{2})} \cdot \frac{(\sqrt{3}+\sqrt{2})}{(\sqrt{3}+\sqrt{2})} = \frac{\sqrt{3}+\sqrt{2}}{9-4} = \frac{\sqrt{3}+\sqrt{2}}{5} \in I$	4
5.4 :	$\frac{1}{(\sqrt{3}-\sqrt{2})} \cdot \frac{(\sqrt{3}+\sqrt{2})}{(\sqrt{3}+\sqrt{2})} = \frac{\sqrt{3}+\sqrt{2}}{3+2} = \frac{\sqrt{3}+\sqrt{2}}{5} \in I$	10
5.5	they did not know	9

6. Edit the expression $\sqrt{a^6b^4 - a^4b^6}$, at which $a, b \geq 0, a > b$.

Answer	Number of responses	
6.1 : $\sqrt{a^6b^4 - a^4b^6} = \sqrt{a^4b^4(a^2 - b^2)} = a^2b^2(a^2 - b^2)^{\frac{1}{2}}$	21	
6.2 : $\sqrt{a^6b^4 - a^4b^6} = \sqrt{a^4b^4(a^2 - b^2)} = a^2b^2\sqrt{(a - b)(a + b)}$	14	
6.3 : $\sqrt{a^6b^4 - a^4b^6} = a^3b^2 - a^2b^3 = a^2b^2(a - b)$	13	
6.4 : $\sqrt{a^6b^4 - a^4b^6} = a^{\frac{6}{2}}b^{\frac{4}{2}} - a^{\frac{4}{2}}b^{\frac{6}{2}} = a^3b^2 - a^2b^3$	4	
6.4 :	they did not know	7

On the chart (Figure 1) we can see the number of correct answers to each question.

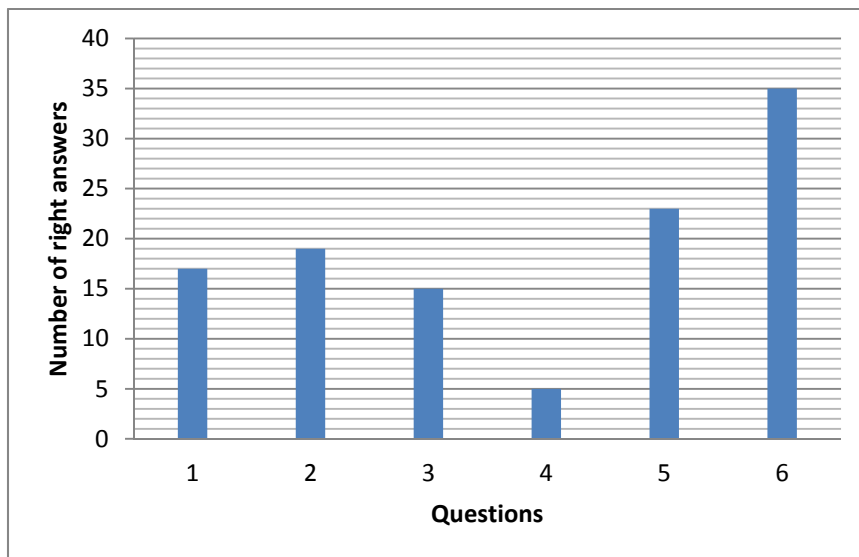


Figure 1: Quantification of right results

Discussion of results

In these responses we identify several overlapping themes: applications of an incorrect or incomplete definition, distraction by a calculator display and lack of awareness of the relationship between fractions and repeating decimals. By testing on understanding irrational numbers we concluded that students relied on decimal representations rather than common fraction representations. We further observed that the connection between two ways to describe irrational numbers - as lacking representation as a ratio and having infinite non - repeating representation - were not well understood. Examples play an important role in learning mathematics. It is hard to imagine learning math without the

consideration of specific examples. Very good way is a “Problem solving”. What is mathematical problem-solving? Mathematical problem - solving is the heart of any mathematicians work and to become a mathematician, one who discovers, conjectures, tests and proves one must become a problem solver. For this to happen, students must therefore engage in solving real problems. Problem solving has a special importance in the study of mathematics. A primary goal of mathematics teaching and learning is to develop the ability to solve a wide variety of complex mathematics problems.

Conclusion

We investigated the understanding of irrational numbers of the group of students at the high school. In this report we focused on the role that representations play in concluding rationality or irrationality of a number. We discovered, that

- the definitions of irrational, as well as rational, numbers were not “active” repertoire of their knowledge;
- there was a tendency to rely on a calculator
- there was a confusion between irrationality and infinite decimal representation, regardless of the structure of this representation

A general suggestion for the teaching practice calls for a tighter emphasis on representations and conclusions that can be derived from considering them. In particular, attending to the connections between decimal and other representations (geometric, symbolic, common fraction, even continued fractions) of a number can be an asset.

Mathematics is a specific subject. To have a number of relevant information, to know the facts or isolated terms, to recall mathematical sentences, definitions is only the first step. The cardinal necessity is to understand their mutual context, which is the only source of true knowledge also applicable in reality.

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