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PRE-SERVICE TEACHERS' PROBLEM POSING IN COMBINATORICS

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ABSTRACT. *Combinatorics is seen as one of the more difficult areas of mathematics to teach and to learn. Mathematical knowledge for teaching combinatorics of 14 pre-service teachers for primary school developed during session integrating mathematical and pedagogical activities was assessed through the problem posing. Knowledge of combinatorics of some was enhanced but not in satisfactory extend. Lack of subject matter knowledge influenced students' ability to pose and subsequently solve combinatorial problems.*

KEY WORDS: *teacher education, combinatorial thinking, mathematical knowledge for teaching*

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Introduction

Discrete mathematics (including combinatorics and graph theory) became part of NTCM standards in United States in 1989 [1]. Since then, the combinatorics is moving to lower grades also in Europe, e.g. in 2004 in Germany (Bayern) [2], in 2007 in Portugal [3]; and since 2008 in Slovakia [4]. Future teachers of mathematics did not experience approach suitable for younger pupils as learners, therefore special focus should be put on this area of mathematics in education of future teachers.

It was shown [5] that when given enough time and hands-on problems, even usually low achieving students can do well in solving combinatorial problems, and vice-versa usually high-achieving students can lose track when dealing with novel problems in combinatorics. Lockwood [6] stressed the role of set of outcomes also in solving of advanced combinatorial problems by tertiary students. She found that students deriving the expression/formula directly from the wording of the task are more likely to overestimate the number of configurations required in task comparing to students first listing a few elements of the set. Hejný [7] claims that development of combinatorial thinking should be based on appropriate amount of combinatorial situations which pupil should deal. By combinatorial situation he understands the triplet: base set (elements inputting to configurations), set of outcomes (set of configurations satisfying the conditions of the task) and organizational principle (structure of set of outcomes in sense of Lockwood). In study [8] it was found that the most efficient verification strategy of students for combinatorial problem is to solve it in other way.

Thus the teacher has to have appropriate knowledge to organize the content and the lesson. Furthermore, he/she should be able to track the process of development for his/her pupils. Jones et al. [9] identified stages in development of combinatorial thinking of children based on SOLO model [10]: Level 1 (Subjective): listing elements in random order, without looking for systematic strategy; Level 2 (Transitional): use of trial-error strategy, discovery of some generative strategies for small sets of outcomes; Level 3 (Informal quantitative): adopting generative strategies for bigger sets or three- and more-

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dimensional situation; Level 4 (Numerical): applying generative strategies and use of formulas; Level 5 (Extended abstract): generalization of relations.

Mathematical Knowledge for Teaching

Issue of teacher knowledge was first arisen in work of Shulman [11]. It was further developed [12, 13] and related to teachers' practice [14] by research team about Deborah L. Ball. Mathematical knowledge for teaching (MKT) is understood as knowledge going behind standard use of mathematics methods, it includes how to represent mathematical concepts and procedures to students, explain mathematical concepts to students and analyze students' solutions and explanations [12].

MKT consists of two domains: subject matter knowledge (SMK) and pedagogical content knowledge (PCK). Both of these are further divided into three subdomains. SMK consists of: (1) Common content knowledge (CCK) refers to general knowledge of mathematics; (2) Specialized content knowledge (SCK) is specific to mathematics teaching. It is used when students' solutions, explanations and reasoning are assessed; (3) Knowledge at the mathematical horizon (KMH) which means relations between concepts and topics included in the mathematics curriculum.

PCK is divided into: (1) Knowledge of content and students (KCS) means understanding students' mathematical thinking; (2) Knowledge of content and teaching (KCT) deals with the ability of teacher to choose and arrange suitable problems for the classroom; and (3) Knowledge of curriculum (KC).

Domains of MKT are related one to each other. Furthermore, "mathematical experiences and pedagogical experiences cannot be two distinct forms of knowledge in teacher education" [15, p. 1964]. Integrated approaches help students to "get a broader view of mathematics, to see its relevance to teaching and to recognize the need for their mathematical and pedagogical development." [15, p. 1964]

Research questions

Can be knowledge in combinatorics developed simultaneously with pedagogical content knowledge?

What kind of knowledge influences primary pre-service teachers' ability of posing combinatorial problems?

The study

Participants of the study were members of international group of 14 pre-service teachers for primary schools taking part in EPTE program attended the session about problem-solving in combinatorics within the mathematic module. The session was led by one of the authors of the paper. We will consider her as participant-observer. The session was audiotaped to enable further analysis. According to [16] "problem posing provides an opportunity to get an insight into natural differentiation of students' understanding of mathematical concepts and processes and to find obstacles in understanding and misunderstandings that already exist", so the data analysis is focused mainly on problem-posing part of the lesson.

Description of the session

The session started with group-work, 14 students formed 5 groups. In the analysis we use notation S*x*G*y* for student *x* from group *y* and T for teacher. Beside problem-solving activities for developing CCK and SCK, students assessed pupils' solutions and pose problem suitable for primary school to connect their subject matter knowledge with pedagogical content knowledge.

The first problem aimed to estimate the level of combinatorial thinking of participating students. It was only two-dimensional: 'Four friends met and shook their hands. How would you describe all handshakes?' The wording of the problem was intentionally formulated to describe the handshakes, to make students to choose the representation of the set of outcomes and not to solve the problem by formula/expression. Only after solving this part, the second question 'How many handshakes there were?' appeared.

Three of five groups were able to solve the problem by employing generative strategy (level 2). Two of successful groups used diagrammatic representation, one group chose a table. Only one student was able to use the formula/expression without hint of the teacher (level 4).

Overall low level of combinatorial thinking of participating students could be seen. Even the presenters from the groups which solved the problem encountered difficulties during the presentation of their own solution. They were not able to explain in detail their employed strategies, particularly why they chose them.

After finishing the frontal discussion about the problem, levels of combinatorial thinking according [9] were introduced. Different students' solutions served as examples for the levels. Second problem [17] was again solved within the group-work. 'How many text messages are sent if four people all send messages to each other? How many text messages are sent with different numbers of people? Approximately how many text messages would travel in cyberspace if everyone in your school took part? Can you think of other situations that would give rise to the same mathematical relationship?' Students were asked to solve this problem by listing elements of set of outcomes, giving it appropriate structure and by formula. All groups were able to solve this problem, mostly by realizing that the number of text messages was twice as much as handshakes in previous problem. After the frontal discussion excerpts of pupils' work also included in [17] was given to students to assess it and possibly formulate an advice which they will give to pupil in their future class if he or she will come with solution like those in the set. They were also asked to come up with good questions which can lead pupils to higher level of solution.

Insufficient level of PCK can be seen in the reflection of one participating students on the session: *Some of them [strategies] were very easy to reproduce, some of them not. Especially when they had a mistake in their strategy and thinking it was not so easy to follow their ideas.*

The third problem was three-dimensional and table or graph was not suitable structure for set of outcomes. 'Peter spends too much time with the computer, so his parents decided to use the password. Peter heard that the first password consists of 4 characters, digits 0 to 3, each only once.' [18] Four groups succeeded in solving this problem, mostly by ordering the strings by their value as numbers. One group was already familiar with tree diagram, which was also presented as possible generative strategy suitable for more-dimensional combinatorial problems. Just after they solved the problem by expression/formula $4 \times 3 \times 2 \times 1$ students realized that this was factorial of number 4. So they activated their CCK, although only on the level of visual recognition, without deeper understanding of the matter. Students' understanding of the importance of generative strategies can be seen: *Of course my chaos system failed and the "tree-system" was quite better to understand.*

The last activity within described session was to design a combinatorial problem suitable for primary school. Students were asked to find the solution by listing elements of set of outcomes, suggest appropriate structure for set of outcomes and, finally, solve the task using the formula/expression. Outcomes of this group-work and following discussion are described and analyzed in following section.

Data analysis

Group 1

Posed problem: Silvia has 5 skirts (yellow, red, green, pink, blue), three pairs of shoes (sneakers, ballerina shoes, slippers) and 2 tights (orange and brown). How many outfits can she create?

Group was able to use the multiplication rule in this problem and estimate the number of outfits as $5 \times 3 \times 2 = 30$. They have chosen tree diagram as the structure for set of outcomes. They posed and solved the problem very quickly and then they discussed possible attitudes how to present the solution in the classroom.

Group 2

Posed problem: The cipher bicycle lock has four digits 0-9. You forgot the password. How many numbers (in worst case) you have to try to unlock it?

The group struggled with the solution. When the teacher came and asked how their work was going, students replied that they got lost. Teacher solved the task in her mind and found out that 104 are too many possibilities for primary level. She thought that students are able to solve the designed task by expression/formula and she wanted them to come up with the task which has set of outcomes of reasonable size.

Teacher (T): *How many possibilities there are? Can you calculate it without writing them all down?*

S1G2: *OK, let's have only numbers 0-4*

There is not clear, whether the student was aware of the solution of the problem or she lowered the number of possible digits based on teacher's question. Then she started to write down (see Table 1):

0,1,2,3	24 possibilities
1,2,3,4	24 possibilities
0,2,3,4	24 possibilities
0,1,3,4	24 possibilities
Total	96 possibilities

Table 1 Excerpt of solution of group 2

S1G2: *I miss some, but I do not know how they should look like.*

T: *Why do you think the numbers cannot repeat?*

Student wrote down the group of four digits

0,0,1,3

S1G2: *How can I find out how many of this kind there are?*

Another student from this group did not participate in discussion with the teacher, but after the first student lowered the size of base set, she tried to solve the problem on her own.

S2G2: *Let it be, we will use the tree.*

Then the group drew one "strand" of the tree diagram, the codes starting by 11. During the presentation they commented on it:

S2G1: *it is obvious how the tree will look like*

S1G1: *and the expression will be 5 times 5 and so on.*

After the two groups presented their problems, teacher wanted the students to compare the two problems, both solved by tree diagram and multiplication rule.

T: *How do the two presented problems differ?*

S1G4: *There are so much other possibilities in the second case.*

S1G1: *I have chosen quite small numbers to have it feasible to draw whole the tree on the blackboard.*

S2G3: *In the first case, you know how many pieces of each kind you have, but in the second task you have to think after each level of the tree.*

S2G1: *Yes, in our case you just multiply the number of things. But in the second task you multiply 5×5×5×5, you have to come up with the next number to multiply.*

Group 3

Posed problem: You have 5 children (Michal, Fero, Juro, Katka and Zuzka) and five pieces of fruit (banana, apple, orange, pear and a kiwi). How many possibilities you have to give children the fruits?

The group struggled with the solution; they tried to find all the permutations of the set of children and all permutations of the set of fruits. One member of group calculated that number of possibilities how to arrange children is 5! = 120, as well as the number of possibilities how to arrange fruits, but she was not able to put the two obtained numbers together. So, they decided to solve the task on lower level.

They started by drawing the diagrams mapping children and fruits together. After 3 diagrams they refused from drawing and started to write down some configurations of ordered pairs. There were app. 20 listed on their paper when teacher came to check the progress in the group. She scanned solution of the group but neither saw any structure of the set, nor knew the wording of the problem to analyze where the trouble came from.

T: *What problem have you posed?*

S1G3: *We have children and we have fruits: banana, orange, apple, pear and kiwi, so we are going to give out the fruits to children.*

Teacher got a bit confused and wanted students to elaborate more the wording of designed problem.

T: *So, you can give bananas to all the children.*

S2G3: *No, you have only one banana.*

T: *Does it mean that each child will get different kind of fruit?*

S1G3: *Yes, exactly.*

T: *And what about the number of children and fruits, are they the same?*

S2G3: *Yes, we have 5 children and 5 pieces of fruits.*

Teacher checked down listed configurations and chosen two that differed only on order, but she was still not sure about the wording of the problem.

Mb, Fa, Jo, Kp, Zk　　　　Fa, Kp, Mb, Zk, Jo

T: *How are these two groups different?*

S1G3: *In the left case, Michal is the first to get fruit, in the right one Fero is the first.*

T: *So, did you include this in your formulation of the problem?*

S1G3: *No, I just want to give children the fruits.*

T: *Does it matter, in your combinatorial situation, who get the fruit first?*

S1G3: *No… so, I can put the fruits in one front and mix the children. Who comes first, will get a banana, the second will get apple.*

T: *Can you continue with your task now?*

S1G3: *There are too many possibilities…* [She looked at their expression to check how many items she will need to avoid too many possibilities]. *I will have only three of the children and the fruits.* [Then she wrote table with all the possibilities]

Students independently realized that listing of 120 possibilities was not good idea for the classroom. Using the mathematical knowledge she chose the appropriate base set
T: *How did you obtain the columns of the table?* (see Table 2)

Banana	Fero	Fero	Michal	Michal	Juro	Juro
Apple	Michal	Juro	Juro	Fero	Michal	Fero
Orange	Juro	Michal	Fero	Juro	Fero	Michal

Table 2: Excerpt of solution of group 3

S1G3: *I chose the first, and then there are only two possibilities for other two. So I write the two remaining children and then just switch their order.*

The system that student chose is not a generative strategy. That wouldn't be usable with many items. So she did not think about it, just adjusted it to get the outcome. On the other side, the student was already aware of the number of possibilities and she could consider that the sophisticated structure was not necessary for such a small set. After being satisfied with the mathematical success, she might not put the priority to solution on lower level, suitable for primary pupils.

Conclusions and discussion

In developing PCK, also CCK has to be taken into account. Students in groups 2 and 3 intuitively underestimated the number of possibilities in cases of variations and permutations what is in accordance with [19]. During process of solving the posed problem both groups lowered the size of base set, but in case of group 2 not enough. Furthermore, group 2 did it only after prompting by the teacher. The difference can be in the level of combinatorial thinking of students. Although group 3 struggled with solution in solving their problem, they got partial result in form of expression/formula. After teacher's intervention in set of outcomes, they were able to solve the posed problem. Group 3 decided about the number of children and fruit by expression/formula, group 2 did not calculate the number of possibilities, even during the presentation they said *you have to just multiply 5 times 5 etc*. Student from another group (group 1) had to remind them with the number. High level of CCK of students in group 1 was also observable in the comment about informed decision when posing their problem. We can assume that without satisfying level of subject matter knowledge students can experience difficulties in activities aiming to pedagogical content knowledge.

The role of CCK, especially in pre-service teacher training, is often underestimated, particularly by students. During problem-posing and subsequent problem-solving of posed problem the importance of mathematical knowledge may emerge also for student-teachers. According to [16] and [20] problem-posing activities are very suitable in training of mathematics teachers. They can uncover possible misconceptions or insufficient knowledge of student-teachers. Combinatorics was not new knowledge for participating students; all the students came up with factorial as a formula for number of permutations after solving the Internet problem by tree diagram. Even after passing courses in mathematics it seem advantageous to include activities developing the mathematical knowledge of student-teachers in courses focused on pedagogies. We can see the shift in the level of combinatorial thinking in the comment of student from group 2 [18] *Of course my chaos system failed and the "tree-system" was quite better to understand*, although even after the shift it was not adequate for future teacher. On the other side, to experience own development as a learner can enhance both, subject matter knowledge and pedagogical content knowledge. This is in accordance with [16], [15] or [21] claiming that mathematical and pedagogical knowledge should be blended to develop mathematical knowledge for teaching.

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