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PUPILS´ SOLUTIONS OF A GEOMETRIC PROBLEM FROM A MATHEMATICAL COMPETITION

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ABSTRACT. *In this article we list a brief analysis of pupils' solutions of a geometric task from the 63th Mathematical Olympiad category Z9. The task was given under regional competition category to the pupils at lower secondary education. We wondered what solutions pupils used and what mistakes occurred in their solutions. We present some solutions of the pupils with specific examples of their sketches.*

KEY WORDS: *competition, analysis, solutions, geometric problem*

CLASSIFICATION: *D54, G44*

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Introduction

According to the viewpoints of many pedagogues as well as researchers from didactics, in order to achieve better results of pupils, it is not enough only to use suitable textbooks or other teaching materials in the teaching process. Pupils must build not only their knowledge, but also an active approach to learning itself. Therefore, for example, problem solving is currently considered a basis for learning.

In PISA study [1] problem solving is defined as an individual ability to use cognitive processes for solving real interdisciplinary problems, when the path to the solution is not immediately visible and the content of knowledge areas, which are necessary to be applied to the solution, is not obvious at first sight.

In the official Slovak document entitled National Program of Education [2] it is also shown that:

- it is necessary to include problematic tasks throughout studies
- the study of mathematics at secondary schools contributes to the development of key competencies for solving of problems, it means: to apply appropriate methods to problem solving which are based on analytical and critical or creative thinking; to be open for capturing and exploiting solving problems with different and innovative practices; to formulate arguments and proofs for defending their results.

As the author claims, in [3] understanding in mathematics, when dealing with also non-mathematical problems and tasks, there is apparently the need for application of mathematical knowledge acquired in non-standard situations. The effort of teachers should encourage students to solve problems in different ways, in regard to their knowledge, skills and acquired mathematical tools.

In this article we list a brief analysis of pupils' solutions of a geometric task from the 63th Mathematical Olympiad category Z9 (the pupils at lower secondary education).

The selected problems from the Mathematical Olympiad and the pupils´ solutions

We have chosen a suitable task of geometry, which was included within the 63th Mathematical Olympiad 2013/2014 as the task of regional competition of category Z9. We wondered what solutions the pupils used and what mistakes occurred in their solutions.

The geometric task: *Within an equilateral triangle ABC there is inscribed an equilateral triangle DEF. Its vertices D, E, F lie on sides AB, BC, AC and the sides of triangle DEF are perpendicular to the sides of triangle ABC (as shown in Figure 1). Also, segment DG is the median of triangle DEF and point H is the intersection of DG, BC. Determine the ratio of triangle HGC to DBE.* [4]

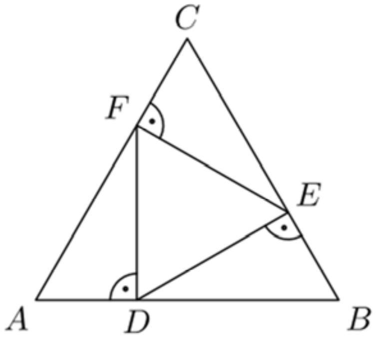

Figure 1: The geometric task

These examples are the solutions of three pupils and the task was solved correctly by 9 of 30 pupils.

<u>The 1st pupil´s solution:</u> In most cases the pupils confirmed that triangle *DEF* is also equilateral, when they added the angles of triangle *ABC*, they found out that the median and height from point *D* of triangle *DEF* are identical. Then they found out that segments AC and DH are parallel (as shown in Figure 2) so $|GH| = \frac{1}{2}|FC| = \frac{y}{2}$, triangles *ADF, BED, CFE* are equal, then$|DB| = |CE|$ and $|DB| = |CE| = |AF| = 2.|HE|$.

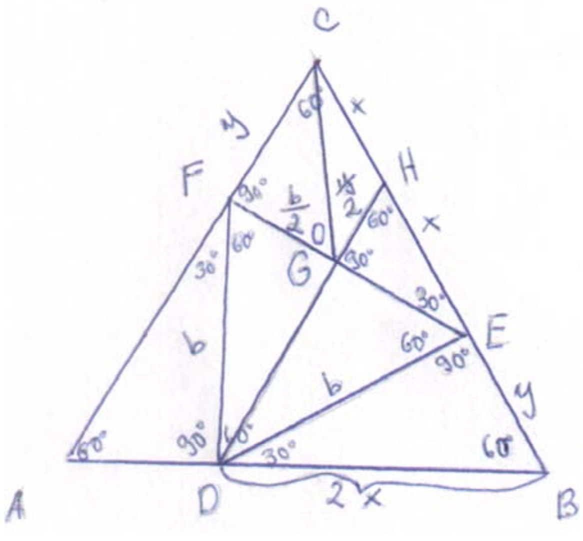

Figure 2: A sketch of a pupil´s solution

They obtained relations for the areas of triangles *FEC, HGE*, *FGC* and *GHC*. Whereas the areas of triangles *CFE* and *BED* are the same, they just put in ratio the areas of triangles *HGC* and *DBE*, which is 1:4.

The 2nd pupil´s solution: The pupil used the following pictures for solving the task (see Figure 3).

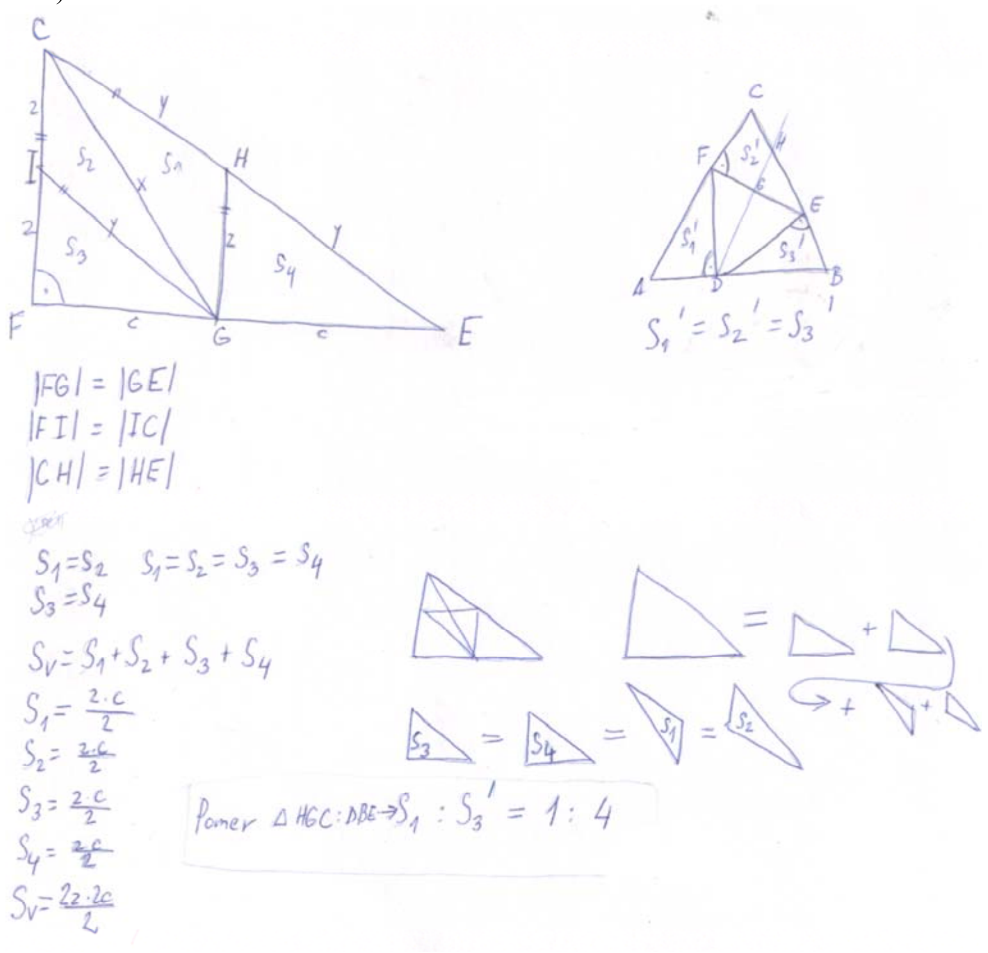

Figure 3: The 2nd pupil´s solution

The 3rd pupil´s solution: Other pupil´s solution contained three pictures. As we can see in Figure 4, beginning of the task solution is identical to the solution of the first pupil in our article.

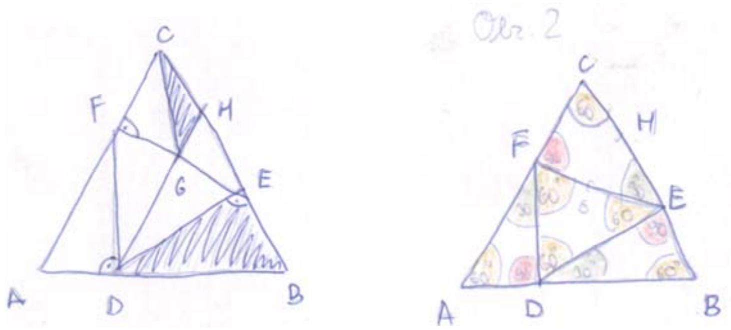

Figure 4: The 3rd pupil solution

Then in Figure 5 there is a detail of triangles *FEC* and *HGC*, so from the existing relations it is valid that: $|HX| = |HG|, |FG| = |GE|$ and the areas of triangles *GEH, HXC* are the same. So the area of quadrilateral *FGXC* is the same as the area of the right triangle *FEC*. The area of triangle *HCX* is equal to a half of the area of triangle *GXC*. Whereas the area of triangle is *GXC* is again a half of the area of the right triangle *FEC*, so the area of the triangle is equal to $\frac{1}{4}$ of its area. Therefore, the ratio of the triangles is $|HGC|:|DBE| = 1:4$.

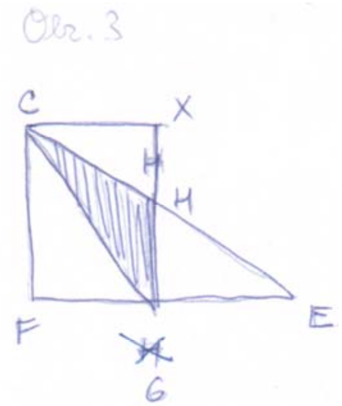

Figure 5: A detail of the triangle

Mistakes in the pupils´ solutions

In this part of the article there is a list of the most frequent mistakes that we have seen in the pupils´ solutions. So incorrect solutions were mainly:

- a graphical solution of the problem – complementing the known facts into pictures (see in Figure 6) followed by the result of the solution,

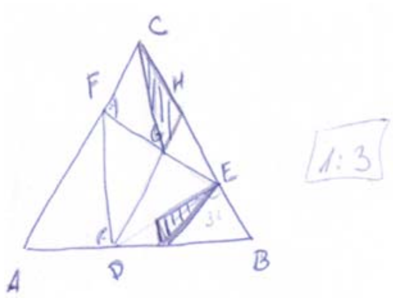

Figure 6: One graphical solution of a pupil

- an incorrect approach in the first step of the solution, for example: segment *DG* is the median of triangle ABC which means $|BE|:|EC| = 1:2$,
- from incorrect facts some got the correct result, for example: in the solution a pupil used the property that the median divided triangle into two equal triangles in the ratio 2:1,
- pupils found that triangles *BED* and *CFE* are equal or determined the size of some angles and then considered the identity of the median and height, but other considerations were not correct,
- searching for the area ratio of trapezoid *CFGH* to triangle *HGE* (3:1), but then the faulty conclusion followed and the pupils also used a fictional length,
- they determined that the right triangles are identical, then they completed the picture, however, without explaining other facts,
- measuring the lengths of the sides and heights of the triangles, they followed with other calculations or finding out of the fact that the right triangles are identical,
- confirmation that triangle *FHC* is an equilateral triangle, then finding out that $|CH| = \frac{1}{3}|CB|$ and the next steps of the solution were wrong,
- the solutions often contain fictional dimensions in the picture and so the next steps were incorrect.

Conclusion

We can observe the fact that when solving the given task, the pupils (who have more than average mathematical skills because they progressed to the mathematical competition) can solve a more demanding geometric task at different levels. The pupils have proven their individual skills in their solutions, individual solving strategies, their registrations or justification.

These results just like the study by [5] confirm the fact that pupils of certain age, assuming the appropriate level of solving and mathematical skills and knowledge, build and develop skills to solve mathematical problems by finding and choosing their own solution strategies.

Therefore, in supporting the work with talented pupils we can see the way to develop pupil´s individuality.

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