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# DIDACTIC EMPATHY OF ELEMENTARY MATHEMATICS TEACHERS

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**ABSTRACT**. *This paper deals with the importance of didactic empathy of elementary mathematics teachers in connection with the current trend of constructivist way of teaching. The authors develop Hejny's method of schema-oriented education and work in the didactical environment Cube Buildings. In this environment, they assign tasks to children aged 5–6, namely to build a cube building according to a scheme and display it in any way in a plane. Before carrying out the research the authors predict children's solutions, both qualitative and quantitative. The qualitative prediction discriminates three basic conditions: correspondence with the scheme, building as a whole, and representation of the cube that is hidden in front view. Other qualitative conditions are observed with the last two conditions. The authors examine quantitative indicators in every qualitative condition defined in this way. Prediction, real data and correspondence rate are processed in tabular form. Based on the results analysis, the authors propose application in the training of future teachers of elementary mathematics.*

**KEY WORDS:** *didactic empathy, scheme building, cube buildings, teacher training*

**CLASSIFICATION:** *D49, D79*

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## Problem formulation and aims of research

Current trend in the teaching of elementary mathematics in the Czech Republic is the schema-oriented education [3]. The method leans on constructivist way of the teaching process with strong emphasis on the pupil's personality, respecting it and developing it [2]. In contrast with the traditional approach to teaching, the significant amount of responsibility for finding and verifying mathematical associations (terms, claims, algorithms, etc.) is shifted from the teacher to the pupil – the pupil is the actor of the process; they are the ones who discover; the teacher coordinates and points in the right direction, presents suitable tasks. Lecturing in the traditional sense is not present. Work with pupil's errors has a significant place; it is an important diagnostic tool for the pupil's comprehension of mathematical situation, not an indicator of their failure. Emphasis is placed on peaceful and friendly teaching environment with mutual trust (between the teacher and the pupil), which is a necessary condition for pupils' discoveries. For this reason, we examine the ability of the teacher to sympathise with the pupils' feelings and thinking in order to optimize their own thinking and acting to a) respect the pupil's personality in its feelings (for good mood in the classroom) and b) develop pupil's personality in thinking (to arrive at mathematical discoveries).

By discovery we understand the shift from "don't know, don't understand, can't" to "know, understand and can", in which the pupil is the main actor. Thus it is a "discovery" on individual level, in connection with the specific pupil. The discovery is accompanied with the feeling of happiness – "the eureka effect", which serves as powerful motivation for further attempts at discoveries. We support that significant pupils' discoveries (first pupil in the class, unconventional solution…) be presented to other pupils, who can then use them for their own discoveries. The important aspect is that it is not the teacher who presents the "discoveries".

All the above mentioned puts increased requirements on the teacher. First of all they must accept their changed pedagogical role, whether they are an active teacher with experience or a pedagogy student, who remembers the traditional teacher's role from their own school years.

The aim of our long term research is to develop and verify tools to support the development of didactic empathy of teachers of elementary mathematics. We work in various didactical environments and utilize different methods. Currently, we have chosen the didactical environment of cube buildings and method of comparing the records. We base our work on the theoretical notion of procept by Gray and Tall, who introduced this term for mathematical objects that can be perceived to signify process and concept at the same time [1]. We expand the theory by amalgam of Hejny, who applies their ideas to geometric environment [3]. We perceive the final structure as concept, its creation as process, and the duality of the concept and the process of building as amalgam. In experiments, we strive to use the amalgam transfer, i.e. the transfer of conceptually described structure to the process and the other way round. We take into account the research results of Krpec in mental schemes of children [4].

**Methodology**

106 pupils aged 5–6 took part in the experiment in two phases: on 6. 2. 2013 it was 21 children from kindergartens from Hlubočec, Pustá Polom and Budišovice; and between 16.–26. 1. 2014 it was 85 children from various kindergartens in the Moravian-Silesian region (total of 20 schools). We worked with each child individually. We placed a cube building in front of the child without their seeing the building process, Fig. 1.

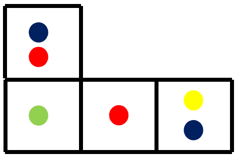

Figure 1: Given cube building

Each child had six coloured cubes at their disposal (see Fig. 1), four colour crayons corresponding with the colours of the cubes and a sheet of A4 sized paper. In the first part of the experiment we asked the child to build the same building they saw. This task was included for the child to experience the amalgam transfer: input concept (provided building) – process (building by a scheme) – output concept (their own building). In the second part of the experiment we asked the child to use crayons to draw the building in such way that somebody else could build it the same way according to their drawing.

Before this experiment we had created a prediction of children's inputs in which we recorded individual ways of solving both parts of the experiment and their percentage proportions

**The presentation of the results and their analysis**

In predicting the first part of the experiment (building by a scheme) we anticipated that all children will build the building correctly. In predicting the second part of the experiment we anticipated that 90% of the children will have three squares next to each other in corresponding colours (from left: green, red & blue), but will differ in representing the other cubes, 5% will use top view (cubes on top of each other will be represented by using two corresponding colours in one square or symbols will be used – lines, pips, smaller squares, etc.) and 5% of the children will not start to solve the task.

As far as the representation of the yellow cube in the first group of children is concerned, from the 90% of children, 90% represents the yellow cube in front view, 10% in top view (draw the yellow colour over the blue one or otherwise mark two colours into one square). Representing the "hidden" red cube in the same group, 70% of the children mark it in top view (Fig. 2a, b, c), 20% do not represent it at all (Fig. 3) and 10% of the children represent the red cube as square (not in front view or top view), Fig. 4. The blue cube is represented depending on the red cube representation either as square "above" its image (20%) or either in front view or top view – this cannot be determined, but we assume that those who do not use top view for the red cube, will not use it for the blue one either, i.e. perceives the representation as front view (80%).

Figures 2a, b, c: Hidden red cube in top view

Figure 3: Hidden red cube is missing

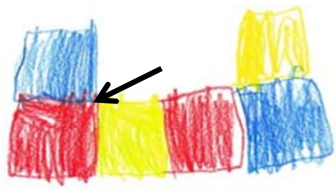

Figure 4: Hidden red cube represented differently

We processed the overview of predictions and results into tables. The **predictions** are presented in both percentages (columns with % headings) and absolute numbers (columns with No. headings), except Table 3 with absolute numbers only; **results** in absolute numbers (columns with No. headings) and the difference between predictions and results in absolute numbers is put on a scale (++ for 1–4 number difference, + for 5–9 number difference, +− for 10–14 number difference, − for 15–19 number difference and −− for 20–30 number difference). The prediction numbers were set using qualified guess. In Table 1 we compare the prediction and results of building by scheme: 13 children did not build the building correctly, the difference between the prediction and results is significant in its quality (the occurrence of incorrect building was not expected at all). Table 2 compares the prediction and results of representing the building as a whole; we consider the difference in the "Not Solving" category significant (all children tried to come up with a solution). Table 3 shows the overview of the predictions and results of representing the yellow, "hidden" red and blue cube in the 2$^{nd}$ upper plane. In this case, we only did a good prediction of the yellow cube representation; with the representation of the red cube we did not expect any other form than square (other image came up 16 times), We expected the children not to represent the cube at all (only 6 occurrences); to put it in front view, or top view (only 48 occurrences); and not to use a square outside the given front view, or top view (31 occurrences). It must be noted that we considered the representation of this cube to be particularly difficult. The representation of the blue cube was dependent on the representation of the "hidden" red cube, but with the exception of top view representation we predicted the numbers better.

The table does not show some specific and interesting solutions of the children, but given the focus of this research, we do not include them here.

The key aim of our research was to do an analysis of expected task solutions (qualitatively: not to omit any solution; quantitatively: to maximally approximate to real results). The key result of our research was the comparison of the prediction with real-life results. We did not omit any solution in the qualitative part; therefore the experiment can be evaluated in the given parameters. In the quantitative part we predicted some parameters wrong to a high degree (--) or wrong (-).

| Building is identical | | | | Building is not identical | | | |
|---|---|---|---|---|---|---|---|
| prediction | | result | | prediction | | result | |
| % | No. | No. | Evaluation | % | Evaluation | No. | Evaluation |
| 100 | 106 | 93 | 0 | 0 | 0 | 13 | 0 |

Table 1: Prediction – building by the scheme

| Three squares next to each other, differences in further representation | | | | Top view with more colours in one square | | | | Not solving | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| prediction | | result | | prediction | | result | | prediction | | result | |
| % | No. | No. | Evaluation | % | No. | No. | Evaluation | % | No. | No. | Evaluation |
| 90 | 96 | 101 | + | 5 | 5 | 5 | ++ | 5 | 5 | 0 | + |

Table 2: Prediction – representing the whole building

| | Front view (F) | | | Top view (T) | | | Square outside of F and T | | | Not represented | | | Rectangle or otherwise | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| cube | pred. | result | | pred. | result | | pred | result | | pred. | result | | pred. | result | |
| | No. | No. | Eval. | No. | No. | Eval. | No. | No. | Eval. | No. | No. | Eval. | No. | No. | Eval. |
| yellow | 91 | 95 | ++ | 10 | 2 | + | 0 | 3 | ++ | 0 | 1 | ++ | 0 | 0 | ++ |
| red | 0 | 5 | + | 71 | 43 | –– | 10 | 31 | –– | 20 | 6 | 0 | 0 | 16 | – |
| blue | 81 | 85 | ++ | 20 | 4 | – | 0 | 7 | + | 0 | 3 | ++ | 0 | 2 | ++ |

Table 3: Prediction of representing the yellow cube, "hidden" red one and the blue one on top of the red one

## Conclusion

Providing and evaluating the prediction in both components (qualitative and quantitative) is a significant process in bolstering didactic competencies of teachers regardless of their success rate (including significant differences between the predictions and results). With a higher number of predicted tasks and reflections on success rates, we expect the improvement of didactic empathy of teachers. This should firstly lead to improved predictions of task solving (more detailed and precise with smaller differences between the predictions and results) and secondly help with complex didactic work (communication, formulating questions, creation and grading of exercises, evaluating solutions…). This would be the subject of further research.

We intend to use this experiment to develop prediction abilities of future teachers of elementary mathematics, who would try to do their own predictions of children's results in the first phase, in the same way we did, and add percentage values of predictions in given categories in the second phase. These categories will be created on the basis of the results of the hereby presented research. The result would be a differentiation set of predictions and results of children for each student, which could be further expanded on.

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