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THE ROLE OF THE GRAPHIC DISPLAY CALCULATOR IN FORMING CONJECTURES ON THE BASIS OF A SPECIAL KIND OF SYSTEMS OF LINEAR EQUATIONS

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ABSTRACT. *A graphic display calculator (GDC) was introduced to mathematical education in the 90s' of the last century. Since then a great deal of scientists and teachers have suggested that this portable device could be applied effectively in the process of teaching and learning mathematics. The aim of this paper is to analyze the process of forming conjectures on the base of some special systems of linear equations in respect of the usage of technology. The researched group consisted of students between the age of 17 and 19, who used GDC as a mandatory device during learning mathematics. The results will be compared with some presented in the paper [1] where one can find different kinds of GDC applications in the process of learning mathematics and the process of generalization with GDC usage analyzed in [4] where visual template tasks were taken into consideration.*

KEYWORDS: *graphic display calculator, mathematics learning, forming conjectures, generalization, International Baccalaureate Diploma Programme*

CLASSIFICATION: *B10, C70, D40, H10, H30.*

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Introduction

The introduction of a graphic display calculator (GDC) into mathematics education began in the early 90s' of the last century. Since then a lot of scientists have examined and evaluated the role of GDC in teaching and learning mathematics as far as different types of mathematical activities are concerned. Throughout years GDC has become an obligatory tool for each student in particular educational programmes. Among them there is International Baccalaureate Diploma Programme which is designed for students aged 16-19. What is worth noticing the International Baccalaureate Organization (IBO), founded in the 1960s', is now a leader in the world education[1]. Moreover, the programme is suitable for conducting the research about using GDC in some aspects of learning mathematics. A lot of the researchers proposed the classification of GDC usage in different activities. For example in paper [1] authors try to answer the question "How do students use GDC to support their mathematics education?" Furthermore, they consider some limitations and constraints of using GDC technology that emerged within the classroom and homework practice. In this paper [1] (p. 151) one can find some patterns and modes of the graphic calculator use.

[1] More information about this programme one can get on www.ibo.org or [5], [7] and [8] .

To quote:

Role of the Graphic Calculator	Description of students Actions
Computational Tool	evaluating a numeral expression, estimating and rounding
Transformational Tool	changing the nature of the task
Data Collection and Analysis Tool	gathering data, controlling phenomena, finding patterns
Visualizing Tool	finding symbolic functions, displaying data, interpreting data, solving equations
Checking Tool	confirming conjectures, understanding multiple symbolic forms

Table 1. Classification of using GDC in different aspects proposed in [DZ].

In paper [4] the process of generalization was considered, with respect of using so called visual patterns, in which on the basis of a prior experience and research, I proposed the scheme of generalization using graphic display calculator.

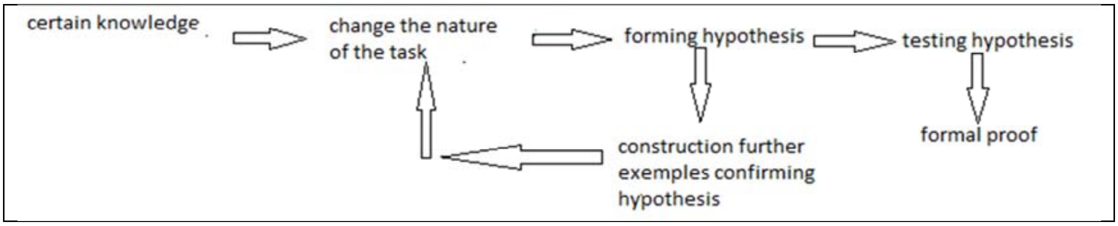

Table 2. The scheme of process of generalization proposed in [4]

The question for the paper is

1. How do the students reason and form conjectures on a basis of a special kind of systems of linear equations?

2. How can the graphic display calculator help students in forming conjectures?

3. Is the process of forming conjectures in this research similar to the process of generalization proposed in [4]?

Methodology and data analysis

In the research students from two different groups of IB class were taken into account. Additionally, all students were taught by me. Furthermore, every student attended the International Baccalaureate Diploma Programme class and all of them started using GDC about 8 months beforehand the research. Prior to the research students were taught about different methods of solving systems of equations (without using any technologies) and they had knowledge about an arithmetic sequence. On the occasion of teaching sequences, students became familiarized with forming conjectures. However, they did not have any experience with working with such exercises as were proposed during the research. In my diagram the described step is called “certain knowledge”.

The data collection was completed in two separated groups of students.

In the first 5-student-group I gave the task[2] and discussed the problem. However, no exact instructions for solving it were provided. During 10-day-period students solved this task individually as their homework without any assistance (with access to GDC). After 10 days students submitted the final version of their solutions. In the second 10-student-group the students were given the task and ordered to solve it during normal lesson time (three consecutive 45-minute-lessons in one day). Similarly as before students did not obtain any special instructions of how to solve the task. During the whole time students worked individually using only sheets of paper, pens and GDCs. After the whole process I interviewed the students whether GDC helped them solve the task. However, only in second group students answered the question in written way. As a teacher I knew the limitations of GDC which could disturb solving this task. Yet I did not inform the students about them. The text of the task given to students is provided below.

Let us consider the 2x2 system of the equation $\begin{cases} x+2y=3 \\ 2x-y=-4 \end{cases}$ Examine the constants in both equations. Solve the system. Create and solve a few more similar 2x2 systems. Make a conjecture and prove it. Extend your investigation to 3x3 systems. Make a conjecture and prove it.

Table 3. The text of the first part of the task used in the research.

This task was not chosen by accident, because a question formulated in such a way can be considered as an open problem which might seem interesting for students (especially for using GDC) and creates an opportunity for experiment and generalization, which is one of the most important purpose of teaching mathematics. Moreover, it enables the observation of different approaches to solving the same problem.

Analysis of students' work

The first question was considerably easy to answer for students. Most of them did not use GDC, and solved it using method of elimination. After analyzing the constants in this systems all students very quickly noticed that coordinates made an arithmetic sequence with different common difference in each equation[3]. They created similar systems using distinct common difference in both equations. Moreover, they quickly realized that all their examples gave the same solutions x=-1 and y=2. For solving further similar systems of equations all students used GDC (mode: EQUA) because, as they commented, not only did they wish to solve them very quickly they also did not want to make any mistakes. As we can see above, students considered systems with distinct common differences in order to check if they would obtain the same result

$\begin{cases} x+4y=7 \\ 4x+2y=0 \end{cases}$	$\begin{cases} -5x-y=3 \\ 6x+5y=4 \end{cases}$	$\begin{cases} 4x+10y=16 \\ 12x+7y=2 \end{cases}$	$\begin{cases} 2x+4y=6 \\ 4x-2y=-8 \end{cases}$	$\begin{cases} \frac{1}{2}x+2y=\frac{7}{2} \\ -2x-\frac{5}{2}y=-3 \end{cases}$

Table 4. Some examples of similar systems proposed by students

2 This task was taken as a part of Portfolio – Internal Assessment for International Baccalaureate Diploma Programme in 2011-2012. In [2] one can find similar system but with different questions. This research was carried out independently.

3 In paper [2] author observed that although their students knew nothing about arithmetic sequence they came to the same conclusion.

In this part students generally used GDC for solving their examples, as they claimed in order to avoid mistakes in calculations and to examine further examples. However, some students preferred methods of elimination instead of using GDC. In my scheme this step is called “change of the nature of the task” .

The next point was concerned with making a conjecture with proof. However, it appeared to be too difficult for students. Only four students from the first group made conjecture properly and proved it. Some of students’ propositions are given below[4]

Let us assume that a is the first term and d is the common difference of the first equation and that b is the first term and kd is the common difference of the second equation.

$$\begin{cases} ax + (a+d)y = a + 2d \\ bx + (b+kd)y = b + 2kd \end{cases}$$

The student did not explain why she used such common differences (where the second one was the multiplication of the first one). Another student proposed a different general pattern for the system.

$$\begin{cases} ax + (a+d)y = a + 2d \\ bx + (b+f)y = b + 2f \end{cases}$$

The third student proposed the following general pattern.

$$\begin{cases} ax + (a+1)y = (a+2) \\ bx + (b-3)y = (b-6) \end{cases}$$

Students at this stage solved their general systems using method of elimination or Cramer’s rule.

It is crucial to notice that each student proposed different general pattern for the system and no one assumed anything in respect to constants. In the first example student used multiplication of the common difference, in the second equation, but in the third example the student used fixed common difference. Only in the second example the generalization was done properly. Nevertheless, nothing was assumed about constants. This step is called “testing hypothesis”. It is worth noticing that students did not do this step well and after forming conjectures they omitted the step “construction of further examples confirming hypothesis”.

In the second group, which worked during the lesson time none of the students generalized this system and no one proposed a formal proof. As they claimed they did not have enough time to do it and they were more focused on producing as many examples as possible to form a conjecture. Similarly, also in this group the step “construction further examples confirming hypothesis” was omitted

The next question of the task concentrated on further examination of systems of type 3x3. Students tried to solve 3x3 systems in which coordinates formed different arithmetic sequences. However, they were disappointed because in each system GDC could not find a solution[5]. What is strange, in comparison to the previous part of the task, students did not use as many examples for forming conjectures. When they examined one or two examples

[4] In the boxes there are parts of original students’ work

[5] Casio model fx-9860GII which students used is not able to solve systems of no solutions and infinitely many solutions

they were quickly discouraged by this part. Although some students tried to solve particular examples using method of elimination, others (especially in the second group) used Cramer's rule where GDC was very helpful in counting needed determinants of matrices (students used RUN-MATH mode for this purpose). In the process of forming conjectures students from the first group used the method of row operations or Cramer's rule. Yet, the same students made the same mistakes in generalization of the system 3x3 as in the generalization of the system 2x2. Two examples of generalized systems are shown below

$\begin{cases} ax+(a+d)y+(a+2d)z=(a+3d) \\ bx+(b+f)y+(b+2f)z=(b+3f) \\ cx+(c+g)y+(c+2g)z=(c+3g) \end{cases}$	$\begin{cases} ax+(a+1)y+(a+2)z=(a+3) \\ bx+(b-3)y+(b-6)z=(b-9) \\ cx+(c+3)y+(c+6)z=(c+9) \end{cases}$

As we can see in the second example the same mistake occured as before (the common difference is fixed in each equation). In conclusion, none of the students from the second group gave general pattern for such a system.

Conclusions

In order to find out a similarity to paper [1] students in this task used GDC as:

- a computational tool (to calculate determinants of matrices in Cramer's rule),
- a transformational tool (to analyze similar systems of equations)
- a visualizing tool (solving systems of equations)

What is important to notice, students did not use the GDC as a checking tool, because as was mentioned before they omitted the step "construction of further examples confirming hypothesis."

To conclude, some problems were confirmed by the research, such as: using GDC in solving problems can encourage students to form conjectures and to produce their own tasks (in this research tasks were similar to proposed by the teacher). Using GDC allowed students to concentrate deeper on the task without worrying about mistakes which could appear during the traditional (paper-pencil) solving of particular systems. Moreover, the students could examine further similar systems in shorter time to observe similarities, what helped them form conjectures. As far as analyzed examples are concerned, students made some mistakes in the process of proving the task. One can conclude that this stage did not facilitate students' thinking or students had a problem with formal proofs. One can think that information about relations between constants in the systems in terms of arithmetic sequence was not important for students.[6]

Students using GDC can work almost as a researcher but GDC does not replace mathematical thinking and does not kill it. Some limitations of GDC (especially when students were not able to solve 3x3 systems which gave infinitely many solutions) show that this device is one of many but not the only device helpful in solving problems. Students noticed these limitations of using GDC but they felt a bit discouraged by using

[6] In paper [2] students did not know anything about arithmetic system but they could solved the problem properly (without generalizations and examining the 3x3 systems).

GDC in the first approach as they tried to find another mode of GDC which could help continue solving the task.

What is important to notice, students (working independently) omitted the step "construction further examples confirming their hypothesis". They formed hypothesis only after the examination of a few examples. After forming the hypothesis they finished the part of the task or (students from the first group only) moved to the next part of the task – formal proof.

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