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DEVELOPMENT OF GEOMETRIC IMAGINATION IN LOWER SECONDARY EDUCATION

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ABSTRACT. *This article approaches the importance and significance of spatial intelligence; moreover it defines areic, geometric imagination and spatial intelligence. Specifically the Tangram brain teaser is suggested as a tool to be used for further development of geometric imagination in lower grade pupils of secondary schools. Pupils shall achieve better understanding of geometry by learning to solve Tangram tasks.*

KEY WORDS: *geometric imagination, Tangram, activities*

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Introduction

Imagination is one of the most important skills in a person's possession and it serves greatly in the versatile development of the pupil's personality. According to several psychologists a developed imagination is closely related to a life success. The imagination is also linked to coping strategies such as the ability to park your car, to place furniture in the room or to orientate in foreign city. Spatial imagination is formed with the help of the real world at an early age of an individual and it is developed along with the sense of the third dimension. The well-developed spatial imagination has a huge value in today's society. Some fields, such as sculpture or topology, would not even exist without developed spatial minds. It has also an important role in science itself where it can serve different purposes. Imagination can be a useful tool, an auxiliary way of thinking, a way to obtain information, a way to formulate tasks or a direct means to solve the problem. By the term of imagination we mean the ability to create images in one's mind, store them in memory and later recall them. [1]. According to [2] the core of spatial intelligence are the capabilities that ensure accurate perception of the visual world and help transform and modify the original perceptions and create own visual experience and conceptual imagination. With those capabilities, we can construct various shapes and manipulate with them. In this article we approach the importance and the significance of spatial intelligence; we define concepts such as areic and spatial imagination and intelligence and suggest ways to develop geometric imagination in lower secondary education through specific tasks, using the Tangram brain teaser.

Geometric imagination in mathematics

Imagination poses an important role in mathematics, and especially in geometry. The notion of geometric imagination is currently used as an ability to imagine:

- geometric figures, their size, position in space and their characteristics,
- given geometric figure in a different position than the original one,
- the change in shape, size, structure and other properties of the figure,
- shapes based on their verbal description,
- shapes by planar image. [3]

The geometric imagination is defined in [4] as the ability to recognize geometric figures and their characteristics, to abstract the geometric attributes of specific objects and to see in them the geometric shapes in pure form; to imagine geometric shapes based on planar patterns in a variety of mutual relations, even those that cannot be demonstrated using physical models. The author states that it is also important to have resources of geometric figures and consequently the ability to recall a variety of forms and shapes and finally the ability to imagine geometric figures and the relationships between them based on their description. The overall layout of graphics on paper, the perception of shape and area, and the relative positions of several plane figures are all related to the areic imagination [5]. Thus, developing areic imagination supports the development of spatial imagination. Spatial imagination is an important part of the child's cognitive competencies. According to [2] spatial ability is divided into three components: the ability to recognize the identity of an object observed from different angles; the ability to imagine movement or change in the internal arrangement of a particular configuration; the ability to think about spatial relations, which are dependent on the orientation of the body of the observer. Solving specific tasks which are aimed at the development of planar imagination can lead to achieving better results in geometry, and solving geometric problems develops spatial imagination. [6] It is important to begin with the development of imagination in early preschool age. There are two favorable periods in the life of a child for the development of imagination: ages between 5-6 years and age of 10-11 years. [7]

Tangram and its use in school curriculum

Brain teaser Tangram is inciting and interesting teaching material that can be used for developing pupils' spatial imagination, logical and creative thinking. It is a convenient combination of didactic game and teaching aid. The word "tangram" means any convex unit divided into seven parts, of which we can compose various figures. Tangram consists of five equilateral right triangles (two large, one medium and two in a smaller size), a square and a rhomboid. Activities with Tangram offer vast possibilities for applying of creative thinking, imagination and creativity. There are many methodological manuals dealing with various tasks and motivational activities with Tangram [1], [6], [8], [9]. According to the State educational program [10] for lower secondary education the theme 'Geometry and measurement' should in parallel develop both areic and spatial imagination and it should also extend pupils' knowledge of geometric figures. According to content and performance standards pupils should understand the characteristics of basic geometric figures, and be able to solve positional and metric problems of ordinary life. An important point here is the development of spatial imagination for which Tangram could be the ideal tool. For this purpose, we have proposed several activities with enhancing level of performance that contribute to the development of geometric and spatial imagination of pupils. We addressed the use of Tangram in the theme 'Area of plane figures' since for this theme there are very little activities, if any. Our suggested activities were carried out at selected schools in Bratislava in March 2014. There were 26 pupils aged 11-12 years (6th grade) in the experimental class. As it is already mentioned above, this period is one of the most suitable for the development of geometric imagination. We took advantage of four lessons of mathematics, which ran within one week. Each pupil drew Tangram in advance, clipped out different geometric figures and painted the figures random color, as we did not have original Tangram for every pupil.

Activities with Tangram for developing geometric imagination

At the beginning we have familiarized pupils with what actually Tangram is and what the basic rules of this puzzle are. Surprising was the finding that none of the pupils knew Tangram yet. In the first stage pupils have been reminded the geometrical figures of which Tangram is composed of and their characteristics. All pupils knew the triangle and square, but they weren't familiar with rhomboid yet. Therefore, we defined a new unit which we presented them as a parallelogram, in which adjacent sides are of unequal lengths and angles are non-right angled.

Figure 1: Pupils when doing Tangram activities

In the second stage, pupils were given the task to assemble the model according to the different shapes of animals, human figures and objects. This task was easily mastered by all the pupils in a relatively short time. Then they got task to put together various images, but in a draft received only the outer figure contour, so they did not know the inner subdivision of composite models (Fig. 2). By compiling these patterns child learns to see the area. Majority of pupils had with this activity problem and few pupils failed to compile the given figure even after several unsuccessful attempts and a sufficiently long time.

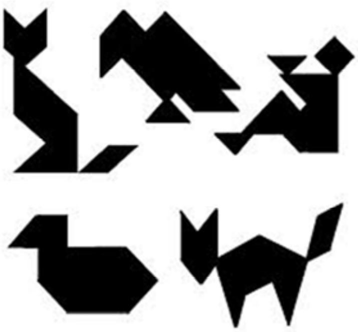

Figure 2: Figures only with outer contour

In the next activity, students were given the task to organize all parts of Tangram into any figure that reminds them of something - an image from ordinary life, which they should additionally name. This activity gives pupils room to apply imagination, originality and creativity and encourages pupils to be active.

 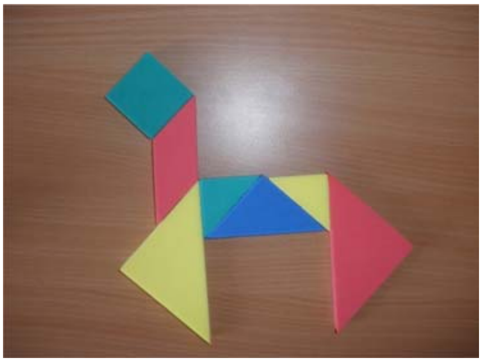

Figure 3: Objects created by pupils – Giraffe and Horse

Next phase we have devoted to activities aimed at the area of the geometrical figures. According to the State educational program [10] pupils should be able to calculate the perimeter and area of the rectangle and square, and also analyze the elements composed of squares and rectangles. Pupils should be also able to design their own methods for calculating the perimeter and the area of geometric figures composed of squares and rectangles. During the verification of these activities pupils have already known to calculate the area of square and rectangle, but they have not yet met with a triangle, which is thematically sorted into a higher grade. Therefore, in this activity we decided that the area of one of the smallest right triangles would be one area unit. Subsequently, we determined the area of the remaining parts of Tangram according to this unit. Some pupils immediately came to the knowledge that square will represent two units of area, since it consists of two identical smallest triangles. To determine area of rhomboid, most pupils had trouble to determine how many units compose its area. Pupils didn't know to combine two smallest triangles in such a way as to give the rhomboid. After some time we determined the area of all remaining parts.

Consequently, pupils should have determined what area has the whole Tangram puzzle. The best pupils of the class after a short while responded with 16 area units. Then they were given the task to build a square and a rectangle, using all parts of Tangram. After our question, which of these two figures has a bigger area, automatically a few pupils answered that rectangle. They absolutely didn't realize that figures composed of exactly the same parts must still have the same area.

In the next activity, pupils were supposed to put together their own figures composed of 12 area units, 14 area units and 15 area units. In this activity pupils should understand that not every figure which has the same area must also have the same shape. Here are a few pupils' works:

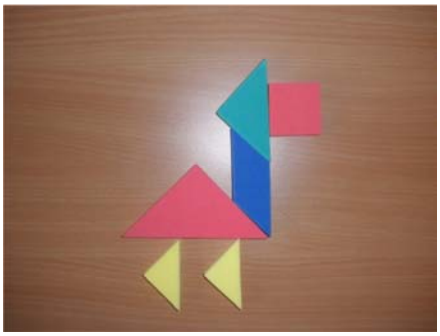 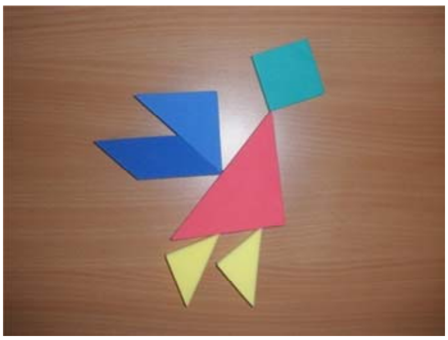

Figure 4: Camel and Cock (12 area units)

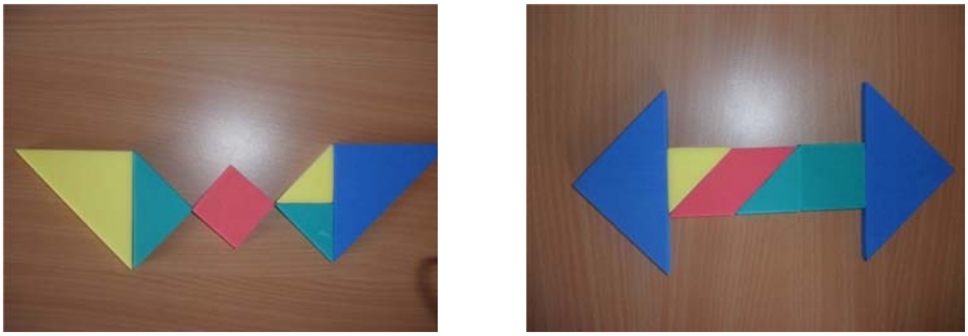

Figure 5: Bat and Arrow (14 area units)

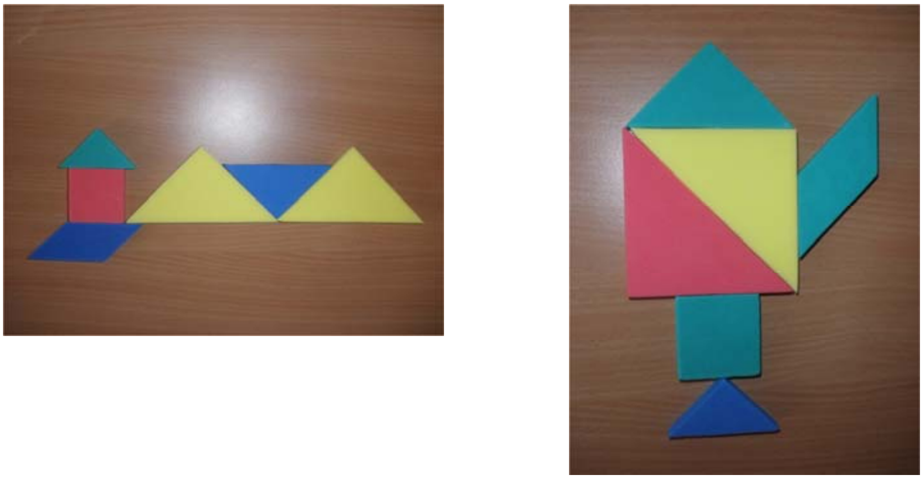

Figure 6: Lake house in the mountains – Fairy tale house (15 area units)

Conclusion

In this article, we have presented specific activities using the brain teaser Tangram, which develop geometric imagination of pupils. We found out that our proposed activities motivate pupils to use their imagination and provide them the opportunity to develop their creative and logical thinking. During the observation of pupils doing the proposed activities, we figured that pupils have lack practical experience with geometry and with changing positions of geometrical figures, which are some of the most important characteristics of geometric imagination. We believe that pupils who are engaged in mathematics by such specific activities have consequently better results in geometry. To confirm our hypothesis, we plan to continue in these activities, with larger pupil sample, we will observe more pupils and compare their results.

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